Problem Set 22

1. (a) **An equation is an algebraic statement consisting of two algebraic expressions connected by an equals sign.**

(b) **A conditional equation is an equation whose truth or falsity depends on the replacement values of the variables within it.**

2. Roots

3. (a), (b), (d)

4. $72 \text{ in.} \times \dfrac{2.54 \text{ cm}}{1 \text{ in.}} = \mathbf{72(2.54)\ cm = 182.88\ cm}$

5. $55 \text{ m} \cdot \text{m} \times \dfrac{100 \text{ cm}}{1 \text{ m}} \times \dfrac{100 \text{ cm}}{1 \text{ m}} = \mathbf{55(100)^2\ cm^2}$
$= \mathbf{550{,}000\ cm^2}$

6.
$$P = 2l + 2w$$
$$124 \text{ cm} = 2(45 \text{ cm}) + 2w$$
$$124 \text{ cm} = 90 \text{ cm} + 2w$$
$$124 \text{ cm} - 90 \text{ cm} = 2w$$
$$34 \text{ cm} = 2w$$
$$w = \frac{34 \text{ cm}}{2}$$
$$w = \mathbf{17\ cm}$$

7.
$$A = \pi r^2$$
$$4\pi \text{ m}^2 = \pi r^2$$
$$4(3.14) \text{ m}^2 = (3.14)r^2$$
$$12.56 \text{ m}^2 = 3.14r^2$$
$$\frac{12.56 \text{ m}^2}{3.14} = r^2$$
$$4 \text{ m}^2 = r^2$$

The radius is **2 m** because $2 \text{ m} \times 2 \text{ m} = 4 \text{ m}^2$.

8. $(-1) - 1 = 0$ $\quad$ $1 - 1 = 0$
$-2 \neq 0$ $\quad$ $0 = 0$

Therefore, **1 satisfies the equation.**

9. $(-3)^2 - (-3) = 12$ $\quad$ $(2)^2 - 2 = 12$
$9 + 3 = 12$ $\quad$ $4 - 2 = 12$
$12 = 12$ $\quad$ $2 \neq 12$

Therefore, **–3 is the root of the equation.**

10. $x^2xxy^2xy^3 = x^2xxxy^2y^3 = \mathbf{x^5y^5}$

11. $a^2aba^3b^2a^5 = a^2aa^3a^5bb^2 = \mathbf{a^{11}b^3}$

12. $p^2m^5ypp^3my^2 = p^2pp^3m^5myy^2 = \mathbf{p^6m^6y^3}$

13. $4p^2x^2kpx^3k^2k = 4p^2pkk^2kx^2x^3 = \mathbf{4p^3k^4x^5}$

14. $-8 - py + 2yp + 4 - y = \mathbf{-4 + py - y}$

15. $m + 4 + 3m - 6 - 2m + mc - 4mc$
$= \mathbf{2m - 2 - 3mc}$

16. $xy - 3xy^2 + 5y^2x - 4xy = \mathbf{-3xy + 2xy^2}$

17. $-3x^2ym + 7x - 5ymx^2 + 16x = \mathbf{-8mx^2y + 23x}$

18. $a(3x - 2) = a(3x) + a(-2) = \mathbf{3ax - 2a}$

19. $4xy(5 - 2a) = 4xy(5) + 4xy(-2a)$
$= \mathbf{20xy - 8axy}$

20. $2x(4a + b - 3m) = 2x(4a) + 2x(b) + 2x(-3m)$
$= \mathbf{8ax + 2bx - 6mx}$

21. $cy(cx - y) = -2(3)[-2(-3) - 3]$
$= -6(6 - 3) = -6(3) = \mathbf{-18}$

22. $|x - a| - a(-x) = |4 - 3| - 3(-4)$
$= 1 + 12 = \mathbf{13}$

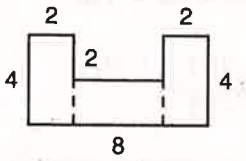
23. $a - b(a^2 - b) = -2 - 3[(-2)^2 - 3]$
$= -2 - 3(4 - 3) = -2 - 3(1) = -2 - 3 = \mathbf{-5}$

24. $(-3)^3 + (-2)^3 - |-2| = -27 - 8 - 2 = \mathbf{-37}$

25. $-3^2 - (-2)^2 + \sqrt[3]{-27} = -9 - 4 - 3 = \mathbf{-16}$

26. $P = \left(4 + 4 + 4 + \dfrac{2\pi(2)}{2}\right) \text{km}$
$= \mathbf{(12 + 2\pi)\ km = 18.28\ km}$

27.



$A = (2 \text{ in.} \times 4 \text{ in.}) + (2 \text{ in.} \times 4 \text{ in.})$
$+ (2 \text{ in.} \times 4 \text{ in.})$
$= 8 \text{ in.}^2 + 8 \text{ in.}^2 + 8 \text{ in.}^2$
$= \mathbf{24\ in.^2}$

28. $\text{Area} = \text{Area}_{\text{triangle}} - \text{Area}_{\text{circle}}$
$= \dfrac{1}{2}(15 \text{ ft})(20 \text{ ft}) - \pi(5 \text{ ft})^2$
$= 150 \text{ ft}^2 - 25\pi \text{ ft}^2$
$= \mathbf{(150 - 25\pi)\ ft^2 = 71.5\ ft^2}$

29. $x = 180 - 115 - 45 = \mathbf{20}$

30. $\text{Volume} = (\text{Area}_{\text{base}})(\text{Height})$
$= \left[\dfrac{1}{2}(9 \text{ in.})(6 \text{ in.})\right](10 \text{ in.})$
$= (27 \text{ in.}^2)(10 \text{ in.})$
$= \mathbf{270\ in.^3}$

Practice 23

a. $x + 5 = 17$

$-5 \quad -5$

$x = \mathbf{12}$

b. $k - 27 = -38$

$+27 \quad +27$

$k = \mathbf{-11}$

c. $x - \frac{1}{2} = \frac{3}{8}$

$+\frac{1}{2} \quad +\frac{1}{2}$

$x = \frac{3}{8} + \frac{1}{2}$

$x = \frac{3}{8} + \frac{4}{8}$

$x = \mathbf{\frac{7}{8}}$

d. $d + 4\frac{1}{7} = 3\frac{1}{6}$

$-4\frac{1}{7} \quad -4\frac{1}{7}$

$d = 3\frac{1}{6} - 4\frac{1}{7}$

$d = \frac{19}{6} - \frac{29}{7}$

$d = \frac{133}{42} - \frac{174}{42}$

$d = \mathbf{-\frac{41}{42}}$

Problem Set 23

1. **To solve an equation means to find the value(s) of the unknown that makes the equation true.**

2. **Two equations are said to be equivalent if every solution of either one of the equations is also a solution of the other equation.**

3. **180°**

4. $150 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = \mathbf{150(100) \text{ cm} = 15{,}000 \text{ cm}}$

5. $116 \text{ in.} \cdot \text{in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}}$

$= \mathbf{116(2.54)^2 \text{ cm}^2 = 748.39 \text{ cm}^2}$

6. $A = lw$

$209 \text{ in.}^2 = (19 \text{ in.})w$

$w = \frac{209 \text{ in.}^2}{19 \text{ in.}}$

$w = \mathbf{11 \text{ in.}}$

7. $A = \pi r^2$

$9\pi \text{ ft}^2 = \pi r^2$

$9(3.14) \text{ ft}^2 = (3.14)r^2$

$28.26 \text{ ft}^2 = 3.14r^2$

$\frac{28.26 \text{ ft}^2}{3.14} = r^2$

$9 \text{ ft}^2 = r^2$

The radius is **3 ft** because $3 \text{ ft} \times 3 \text{ ft} = 9 \text{ ft}^2$.

8. $x - 4 = 10$

$+4 \quad +4$

$x = \mathbf{14}$

9. $x + \frac{1}{5} = -\frac{1}{10}$

$-\frac{1}{5} \quad -\frac{1}{5}$

$x = -\frac{1}{10} - \frac{1}{5}$

$x = -\frac{1}{10} - \frac{2}{10}$

$x = \mathbf{-\frac{3}{10}}$

10. $x + 1\frac{1}{4} = -\frac{5}{8}$

$-1\frac{1}{4} \quad -1\frac{1}{4}$

$x = -\frac{5}{8} - 1\frac{1}{4}$

$x = -\frac{5}{8} - \frac{5}{4}$

$x = -\frac{5}{8} - \frac{10}{8}$

$x = \mathbf{-\frac{15}{8}}$

11. $(-2) + 2 = 0 \qquad 2 + 2 = 0$

$0 = 0 \qquad 4 \neq 0$

Therefore, –2 satisfies the equation.

12. $(-2)^2 - 2(-2) = 3 \qquad (3)^2 - 2(3) = 3$

$4 + 4 = 3 \qquad 9 - 6 = 3$

$8 \neq 3 \qquad 3 = 3$

Therefore, **3 is a root of the equation.**

13. $x^2ym^5x^2y^4 = m^5x^2x^2yy^4 = \mathbf{m^5x^4y^5}$

14. $x^3y^2myxm = mmx^3xy^2y = \mathbf{m^2x^4y^3}$

15. $xxx^2yyy^3xy = xxx^2xyyy^3y = \mathbf{x^5y^6}$

16. $-4 + 7x - 3x - 5 + 2x - 4x = \mathbf{-9 + 2x}$

17. $p^2xy - 3yp^2x + 2xp^2y - 5 = \mathbf{-5}$

18. $4x(a + 2b) = 4x(a) + 4x(2b) = \mathbf{4ax + 8bx}$

19. $(2x + 4)3 = (2x)3 + (4)3 = \mathbf{6x + 12}$

20. $4px(my - 3ab) = 4px(my) + 4px(-3ab)$
$= \mathbf{4mypx - 12abpx}$

21. $x(x - y) - y = -2(-2 - 3) - 3$
$= -2(-5) - 3 = 10 - 3 = \mathbf{7}$

22. $(a - b)(b - x) = [3 - (-3)](-3 - 2)$
$= (3 + 3)(-5) = (6)(-5) = \mathbf{-30}$

23. $x^3 - a(a - b) = (-2)^3 - 3(3 - 3)$
$= -8 - 3(0) = \mathbf{-8}$

24. $-3^2 - (-3)^2 + \sqrt[5]{32} = -9 - (9) + 2$
$= -18 + 2 = \mathbf{-16}$

25. $\dfrac{-3 - 4(4) - 6}{10 + (-5)(-4)} = \dfrac{-3 - 16 - 6}{10 + 20} = \dfrac{-25}{30} = \mathbf{-\dfrac{5}{6}}$

26. $P = \left(\dfrac{2\pi(6)}{2} + 13 + \dfrac{2\pi(6)}{2} + 13\right)$ yd
$= \mathbf{(26 + 12\pi)\ yd = 63.68\ yd}$

27. $A = (25\text{ cm} \times 8\text{ cm}) + \dfrac{1}{2}(25\text{ cm} \times 10\text{ cm})$
$= 200\text{ cm}^2 + 125\text{ cm}^2 = \mathbf{325\text{ cm}^2}$

28. $3\dfrac{3}{7} \times 3\dfrac{1}{16} = \dfrac{24}{7} \times \dfrac{49}{16} = \dfrac{21}{2} = \mathbf{10\dfrac{1}{2}}$

29. $\dfrac{3\frac{1}{5}}{-2\frac{2}{15}} = \dfrac{\frac{16}{5}}{-\frac{32}{15}} = \dfrac{16}{5} \times \dfrac{-15}{32} = \dfrac{-3}{2} = \mathbf{-1\dfrac{1}{2}}$

30. $S.A. = 2(\text{Area}_{\text{front}}) + 2(\text{Area}_{\text{top}}) + 2(\text{Area}_{\text{side}})$
$= 2(2\text{ m} \times 2\text{ m}) + 2(2\text{ m} \times 2\text{ m})$
$+ 2(2\text{ m} \times 2\text{ m})$
$= 2(4\text{ m}^2) + 2(4\text{ m}^2) + 2(4\text{ m}^2)$
$= 8\text{ m}^2 + 8\text{ m}^2 + 8\text{ m}^2$
$= \mathbf{24\text{ m}^2}$

Practice 24

a. $\dfrac{3}{5}x = 27$

$\dfrac{\frac{3}{5}x}{\frac{3}{5}} = \dfrac{27}{\frac{3}{5}}$

$x = \dfrac{27}{\frac{3}{5}}$

$x = 27 \cdot \dfrac{5}{3}$

$x = \mathbf{45}$

b. $3\dfrac{1}{5}y = 32$

$\dfrac{3\frac{1}{5}y}{3\frac{1}{5}} = \dfrac{32}{3\frac{1}{5}}$

$y = \dfrac{32}{\frac{16}{5}}$

$y = 32 \cdot \dfrac{5}{16}$

$y = \mathbf{10}$

c. $\dfrac{x}{\frac{1}{4}} = 20$

$\dfrac{\frac{1}{4}x}{\frac{1}{4}} = 20 \cdot \dfrac{1}{4}$

$x = 20 \cdot \dfrac{1}{4}$

$x = \mathbf{5}$

d. $\dfrac{x}{2\frac{1}{4}} = 5$

$\dfrac{2\frac{1}{4}x}{2\frac{1}{4}} = 5 \cdot 2\dfrac{1}{4}$

$x = 5 \cdot \dfrac{9}{4}$

$x = \mathbf{\dfrac{45}{4}}$

Problem Set 24

1. **Positive number**

2. **(b), (c)**

3. (a) **60°**

(b) **60°**

4. $280 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = \mathbf{280(5280)(12) \text{ in.}}$

$= \mathbf{17{,}740{,}800 \text{ in.}}$

5. $45 \text{ ft} \cdot \text{ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = \mathbf{45(12)^2 \text{ in.}^2}$

$= \mathbf{6480 \text{ in.}^2}$

6. $A = \frac{1}{2}(24 \text{ cm})(18 \text{ cm}) = \mathbf{216 \text{ cm}^2}$

7.
$$C = 2\pi r$$
$$12\pi \text{ m} = 2\pi r$$
$$12(3.14) \text{ m} = 2(3.14)r$$
$$37.68 \text{ m} = 6.28r$$
$$r = \frac{37.68 \text{ m}}{6.28}$$
$$r = \mathbf{6 \text{ m}}$$

8.
$$x + 5 = 7$$
$$-5 \quad -5$$
$$x = \mathbf{2}$$

9.
$$y - 3 = 2$$
$$+3 \quad +3$$
$$y = \mathbf{5}$$

10.
$$x - \frac{1}{4} = \frac{7}{8}$$
$$+\frac{1}{4} \quad +\frac{1}{4}$$
$$x = \frac{7}{8} + \frac{1}{4}$$
$$x = \frac{7}{8} + \frac{2}{8}$$
$$x = \mathbf{\frac{9}{8}}$$

11.
$$y - \frac{1}{2} = -2\frac{1}{2}$$
$$+\frac{1}{2} \quad +\frac{1}{2}$$
$$y = -2\frac{1}{2} + \frac{1}{2}$$
$$y = -\frac{5}{2} + \frac{1}{2}$$
$$y = -\frac{4}{2}$$
$$y = \mathbf{-2}$$

12.
$$x + \frac{1}{2} = 2\frac{1}{5}$$
$$-\frac{1}{2} \quad -\frac{1}{2}$$
$$x = 2\frac{1}{5} - \frac{1}{2}$$
$$x = \frac{11}{5} - \frac{1}{2}$$
$$x = \frac{22}{10} - \frac{5}{10}$$
$$x = \mathbf{\frac{17}{10}}$$

13.
$$2x = 20$$
$$\frac{2x}{2} = \frac{20}{2}$$
$$x = \mathbf{10}$$

14.
$$3x = 4\frac{1}{2}$$
$$\frac{3x}{3} = \frac{4\frac{1}{2}}{3}$$
$$x = \frac{\frac{9}{2}}{3}$$
$$x = \frac{9}{2} \cdot \frac{1}{3}$$
$$x = \mathbf{\frac{3}{2}}$$

15.
$$\frac{x}{3} = 5$$
$$3 \cdot \frac{x}{3} = 5 \cdot 3$$
$$x = \mathbf{15}$$

16. $\dfrac{x}{\frac{1}{2}} = 4$

$\dfrac{\frac{1}{2}x}{\frac{1}{2}} = 4 \cdot \dfrac{1}{2}$

$x = \mathbf{2}$

17. $(-5) - 5 = 0 \qquad 5 - 5 = 0$

$-10 \neq 0 \qquad 0 = 0$

Therefore, **5 satisfies the equation.**

18. $m^2xyp^2x^3y^5 = m^2p^2xx^3yy^5 = \mathbf{m^2p^2x^4y^6}$

19. $3p^2xxypp^3xy^2 = 3p^2pp^3xxxyy^2 = \mathbf{3p^6x^3y^3}$

20. $a + 3 - 2a - 5a + 5 - a = 8 - 7a = \mathbf{-7a + 8}$

21. $-3x^2ym + 5myx^2 - 2my^2x = \mathbf{2mx^2y - 2mxy^2}$

22. $(3a - 5p)xy = (3a)xy + (-5p)xy = \mathbf{3axy - 5pxy}$

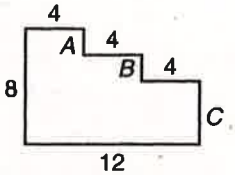
23. $(-x)^3 - y = [-(-2)]^3 - 4 = 2^3 - 4 = 8 - 4$
$= \mathbf{4}$

24. $a^3 - (a - b) + |a - b|$
$= (-2)^3 - (-2 - 3) + |-2 - 3|$
$= -8 - (-5) + |-5| = -8 + 5 + 5 = \mathbf{2}$

25. $-2\{-5(-3 + 4) - [3 - (-4)]\}$
$= -2[-5(1) - (3 + 4)] = -2(-5 - 7)$
$= -2(-12) = \mathbf{24}$

26. $(-2)^3 - (-2)^2 - 5 + \sqrt[3]{-64} = -8 - 4 - 5 - 4$
$= \mathbf{-21}$

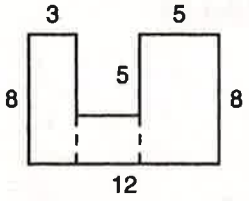
27.



$A + B + C = 8$

$P = 12 + 8 + 4 + A + 4 + B + 4 + C$
$= 12 + 8 + 4 + 4 + 4 + (A + B + C)$
$= 12 + 8 + 4 + 4 + 4 + 8$
$= \mathbf{40\ km}$

28.



$A = (3\text{ in.} \times 8\text{ in.}) + (5\text{ in.} \times 8\text{ in.})$
$+ (3\text{ in.} \times 4\text{ in.})$
$= 24\text{ in.}^2 + 40\text{ in.}^2 + 12\text{ in.}^2$
$= \mathbf{76\ in.^2}$

29. $\text{Area} = \text{Area}_{\text{triangle}} - \text{Area}_{\text{circle}}$
$= \frac{1}{2}(16\text{ ft})(13\text{ ft}) - \pi(4\text{ ft})^2$
$= 104\text{ ft}^2 - 16\pi\text{ ft}^2$
$= \mathbf{(104 - 16\pi)\ ft^2 = 53.76\ ft^2}$

30. $V = (\text{Area}_{\text{base}})(\text{Height})$
$= (95\text{ cm}^2)(16\text{ cm}) = \mathbf{1520\ cm^3}$

Practice 25

a. $\dfrac{2}{5}x - \dfrac{3}{10} = \dfrac{1}{2}$

$+\dfrac{3}{10} \quad +\dfrac{3}{10}$

$\dfrac{2}{5}x = \dfrac{8}{10}$

$\dfrac{5}{2} \cdot \dfrac{2}{5}x = \dfrac{8}{10} \cdot \dfrac{5}{2}$

$x = \dfrac{4}{2}$

$x = \mathbf{2}$

b. $2\frac{1}{4}x + \dfrac{3}{7} = \dfrac{5}{14}$

$-\dfrac{3}{7} \quad -\dfrac{3}{7}$

$2\frac{1}{4}x = -\dfrac{1}{14}$

$\dfrac{9}{4}x = -\dfrac{1}{14}$

$\dfrac{4}{9} \cdot \dfrac{9}{4}x = -\dfrac{1}{14} \cdot \dfrac{4}{9}$

$x = \mathbf{-\dfrac{2}{63}}$