LESSON 10 Division by Zero • Exchange of Factors in Multiplication • Conversions of Area

10.A division by zero

The operation of division is the inverse operation of the operation of multiplication, for division is defined as the process that will *undo* multiplication. Thus if $3 \times 2 = 6$, it is necessary that

$$\frac{6}{3} = 2 \qquad \text{and} \qquad \frac{6}{2} = 3$$

We will now use the same thought process to try to decide what the result will be if we divide a nonzero number by zero. We will use the example of 6 divided by 0.

$$\frac{6}{0} = ?$$

Since we say that division *undoes* multiplication, the multiplication that is to be undone by the above division must be

$$6 = ? \cdot 0$$

But the product of zero and any real number is zero—it is not 6. There is no number that we can substitute for ? so that the product of ? and 0 equals 6. Therefore, we say that since the multiplication does not exist that is to be undone by the division, the expression

$$\frac{6}{0}$$

has no meaning, or is **undefined.** A similar reasoning process is used to show that we cannot divide zero by zero, and we say that zero divided by zero is **indeterminate** rather than saying it is undefined. Thus indicated divisions such as

$$\frac{0}{0} \qquad \frac{142}{0} \qquad \frac{6}{0} \qquad \frac{-5}{0}$$

have no meaningful simplifications.

example 10.1 Evaluate: (a) $\dfrac{4-2-2}{13}$ (b) $\dfrac{13}{4-2-2}$

solution (a) First we simplify the numerator.

$$\frac{4-2-2}{13} = \frac{0}{13} = \mathbf{0}$$

Zero over 13 is read as "zero thirteenths" and has a value of **zero. If the bottom of a fraction is not zero and the top is zero, the fraction equals zero.**

(b) $\dfrac{13}{4-2-2} = \dfrac{13}{0} = ?????$

The expression 13 over 0 has no meaning and thus has no value. **It does not have a value of infinity. It does not have a value of zero. The expression is undefined.**

10.B exchange of factors in multiplication

In Lesson 5, we noted that we can change the order in which signed numbers are added without changing the answer. This is called the commutative property for addition. Now we note that the order of multiplying signed numbers does not affect the answer. This is called the **commutative property for multiplication.**

> COMMUTATIVE PROPERTY FOR MULTIPLICATION
>
> The order in which two real numbers are multiplied does not affect the product. For example,
>
> $4 \cdot 3 = 12$ and $3 \cdot 4 = 12$

The order in which signed numbers are multiplied does not affect the value of the product! This property can be used to show that any number of numbers can be multiplied in any order and the answer will be the same every time.

example 10.2 Find the product: $-4(3)(-6)(-2)$

solution

$-4(3)(-6)(-2)$	given
$-12(-6)(-2)$	multiplied -4 by 3
$72(-2)$	multiplied -12 by -6
-144	multiplied 72 by -2

In the first step, we multiplied -4 by 3 to get -12. In the second step, we multiplied -12 by -6 to get $+72$, which we multiplied by -2 to get the final result of -144.

example 10.3 Find the product: $-6(-2)(3)(-4)$

solution

$-6(-2)(3)(-4)$	given
$12(3)(-4)$	multiplied -6 by -2
$36(-4)$	multiplied 12 by 3
-144	multiplied 36 by -4

example 10.4 Find the product: $-6(-4)(3)(-2)$

solution

$-6(-4)(3)(-2)$	given
$24(3)(-2)$	multiplied -6 by -4
$72(-2)$	multiplied 24 by 3
-144	multiplied 72 by -2

In each of the three preceding examples, the same factors were multiplied, but the order of multiplication was different. The product was -144, however, regardless of the order of the factors.

10.C conversions of area

In this section we will use unit multipliers to convert area measurements.

example 10.5 Use two unit multipliers to convert 44 square inches to square centimeters.

solution We will write 44 in.^{2} as 44 in. · in. Therefore, we have

$$44 \text{ in.} \cdot \text{in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} = \mathbf{44(2.54)(2.54)cm^2}$$

example 10.6 Use four unit multipliers to convert 125 square centimeters to square feet.

solution We will write 125 cm^2 as 125 cm · cm. Therefore, we have

$$125 \text{ cm} \cdot \text{cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{\mathbf{125}}{\mathbf{(2.54)(2.54)(12)(12)}} \mathbf{ft^2}$$

practice Simplify:

a. $\dfrac{-3 - 2}{-2 + 8 - 6}$

b. $\dfrac{-8 + 6 + 2}{8 - 4 - 4}$

c. $-(-4)(-1)(-4)$

d. $2(-6)(10)(-2)$

e. Use two unit multipliers to convert 44 square miles to square feet.

f. Use two unit multipliers to convert 3500 square centimeters to square meters.

problem set 10

1. (10) What property of multiplication states that the order in which two real numbers are multiplied does not affect the product?

2. (9) (a) What operation is the inverse operation of addition?

(b) What operation is the inverse operation of subtraction?

(c) What operation is the inverse operation of multiplication?

(d) What operation is the inverse operation of division?

3. (6,7) (a) What is the additive inverse of $\frac{1}{3}$?

(b) What is the additive inverse of $-\frac{1}{3}$?

(c) What is the sum of a real number and its additive inverse?

4. (6) What is another name for the additive inverse of a number?

Simplify:

5. (10) $-2(3)(4)$

6. (10) $-4(3)(-2)$

7. (10) $4(-3)(-4)$

8. (10) $\dfrac{-2 + 3}{4 - 5 + 3}$

9. (10) $\dfrac{4 + 7 - 6}{2 + 7 - 3}$

10. (10) $\dfrac{-3 + 6 - 1}{-2 + 4 - 2}$

11. (4) Use one unit multiplier to convert 50 inches to centimeters.

12. (10) Use two unit multipliers to convert 48 square inches to square centimeters.

13. (3) The perimeter of a rectangle is 28 centimeters. Two of the sides are each 9 centimeters long. What is the length of each of the other two sides?

14. (8) The length of a rectangle is 15 meters. The width of the rectangle is 8 meters. Find the area of the rectangle.

15. (3) The diameter of a circle is 16 inches. Find the circumference of the circle.

16. (8) The radius of a circle is 6 feet. Find the area of the circle.

Simplify:

17. (9) $\frac{-8}{2}$

18. (9) $\frac{9}{-3}$

19. (7) $3 - (-4) + (-3) - (-4)$

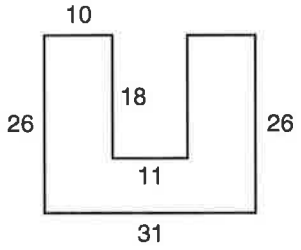
20. (7) $-[-(-4)] - (-3) + 2$

21. (6) $-|-3 - 2| + (-5)$

22. (7) $-\{-[-(-2)]\} - |-4 - 2|$

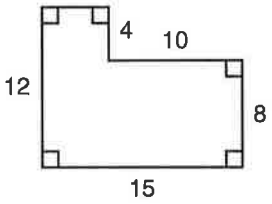
23. (7) $3 - |-2 - 3| + (-6) - (-3)$

24. (3) Find the perimeter of this figure. All angles are right angles. Dimensions are in yards.

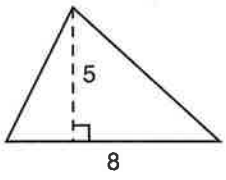


Find the area of each figure. Dimensions are in centimeters.

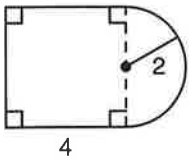
25. (8)



26. (8)



27. (8)



Simplify:

28. (4) $\dfrac{3\frac{3}{5}}{2\frac{7}{10}}$

29. (4) $0.008484 \div 0.0028$

30. (1) The length of $\overline{XY}$ is $3\frac{8}{21}$ meters. The length of $\overline{YZ}$ is $3\frac{2}{7}$ meters. Find XZ.

