LESSON 11 *Reciprocal and Multiplicative Inverse • Order of Operations • Identifying Multiplication and Addition*

11.A reciprocal and multiplicative inverse

If one fraction is the inverted form of another fraction, each of the fractions is said to be the **reciprocal** of the other fraction.

$$\frac{2}{3} \text{ is the reciprocal of } \frac{3}{2}$$

$$\frac{3}{2} \text{ is the reciprocal of } \frac{2}{3}$$

$$-\frac{4}{11} \text{ is the reciprocal of } -\frac{11}{4}$$

$$-\frac{11}{4} \text{ is the reciprocal of } -\frac{4}{11}$$

Since numbers such as 4 can also be written in a form such as $\frac{4}{1}$, these numbers also have reciprocals.

$$\frac{1}{4} \text{ is the reciprocal of } 4$$

$$4 \text{ is the reciprocal of } \frac{1}{4}$$

$$-5 \text{ is the reciprocal of } -\frac{1}{5}$$

$$-\frac{1}{5} \text{ is the reciprocal of } -5$$

The number zero does not have a reciprocal because if we try to write the reciprocal of zero we get

$$\frac{1}{0} \quad \text{(meaningless)}$$

which we say is a meaningless notation because division by zero is undefined. **Zero is the only real number that does not have a reciprocal.** The reciprocal of a number is often called the **multiplicative inverse** of the number.

> DEFINITION OF RECIPROCAL OR MULTIPLICATIVE INVERSE
>
> For any nonzero real number a, the reciprocal, or multiplicative inverse, of the number is $\frac{1}{a}$.

If a number is multiplied by its reciprocal (its multiplicative inverse), the product is the number 1. Thus

$$4 \cdot \frac{1}{4} = 1 \qquad -5 \cdot \frac{1}{-5} = 1 \qquad \text{and} \qquad -\frac{1}{13} \cdot (-13) = 1$$

This simple fact is of great importance and will be very useful in the solutions of equations, a topic that will be discussed in later lessons.

11.B order of operations

If we wish to compute the value of

$$4 + 3 \cdot 2$$

we have a problem. It appears that there are two possible solutions.

(a) $4 + 3 \cdot 2$

Here we will first multiply 3 by 2 to get

$$4 + 6$$

and then add to get 10.

$$4 + 6 = 10$$

(b) $4 + 3 \cdot 2$

Here we will first add 4 and 3 to get 7

$$7 \cdot 2$$

and then multiply to get 14.

$$7 \cdot 2 = 14$$

We worked the problem two ways and got two different answers. **Neither way is necessarily more correct than the other, but since there are two possible ways to work the problem mathematicians have found it necessary to agree on one way so that everyone will get the same answer. They have agreed to do the multiplications first and then to do the additions.** Thus, to simplify an expression such as

$$4 \cdot 3 + 5 - 6 + 4 - 3 \cdot 5 + 6 - 4 \cdot 2$$

we will use a two-step process. First we will perform all the multiplications and get

$$12 + 5 - 6 + 4 - 15 + 6 - 8$$

Now we will do the algebraic additions.

$17 - 6 + 4 - 15 + 6 - 8$	added 12 and 5
$11 + 4 - 15 + 6 - 8$	added 17 and –6
$15 - 15 + 6 - 8$	added 11 and 4
$0 + 6 - 8$	added 15 and –15
-2	added 6 and –8

example 11.1 Simplify: $4 \cdot 3 + 2 + (-3)5$

solution We perform the multiplications first and get

$$12 + 2 - 15$$

which we now add algebraically.

$$12 + 2 - 15 = \mathbf{-1}$$

example 11.2 Simplify: $-5(2) - 3 + 6(3)$

solution We perform the two multiplications first

$$-10 - 3 + 18$$

and now add to get **5.**

example 11.3 Simplify: $4 \cdot 3 - 2 \cdot 5 + 6 - 5 \cdot 2$

solution

$12 - 10 + 6 - 10$	performed multiplications
$\mathbf{-2}$	added algebraically

11.C identifying multiplication and addition

When confronted with an expression such as

$$4 - 3(5) - 7(-6) - (4)(-5)$$

the beginner often has difficulty telling whether quantities are to be added or multiplied. There is an easy way to identify indicated multiplication. **If there is no + or − sign between symbols, multiplication is indicated.** Let us simplify the expression above from left to right by first performing the indicated multiplications. The first place where there is no sign between the symbols is between the 3 and the parentheses enclosing the 5. The second place is between the 7 and the parentheses enclosing the −6, and the third place is between the parentheses enclosing both the 4 and the −5.

$$4 - 3(5) - 7(-6) - (4)(-5)$$



The places where multiplication is indicated are designated by arrows. If we perform the indicated multiplications, we have

$$4 - 15 - (-42) - (-20)$$

Now we can simplify this expression and add.

$$4 - 15 + 42 + 20 = 51$$

practice Simplify. Remember that multiplication is done before addition.

a. $6 \cdot 3 - 4(5)(6)$

b. $3 \cdot 5 + 2 + 4(-2)$

c. $2 \cdot 4 - 3 \cdot 2 - 7 + 5 \cdot 2$

d. $13 - 4(-5) - 3(10)$

problem set 11

1. (11) (a) What is the reciprocal of 2?

(b) What is the reciprocal of −2?

(c) What is the product of any nonzero real number and its reciprocal?

2. (11) What is another name for the reciprocal of a number?

3. (11) Which real number does not have a reciprocal and why?

4. (9) Is the product of two negative numbers always a positive number?

5. (2) What is an acute angle?

6. (2) What is an obtuse angle?

7. (4) Use one unit multiplier to convert 25 meters to centimeters.

8. (10) Use two unit multipliers to convert 40 square meters to square centimeters.

9. (3) The perimeter of a square is 49 inches. What is the length of one side of the square?

10. (8) The area of a square is 16 square feet. What is the length of one side of the square?

11. (8) The diameter of a circle is 14 yards. Find the area of the circle.

Simplify. Remember that multiplication is done before addition.

12. (11) $6 - 8 + 2(3)$

13. (11) $-2 - 3(+6)$

14. (11) $3 - 2 \cdot 4 + 3 \cdot 2$

15. (11) $-3(-2)(-3) - 2$

16. (11) $-4(-3) + (-2)(-5)$

17. (11) $-2 - 2(-2) + (-2)(-2)$

18. (11) $(-5) - (-5) + 2(-2) + 4$

19. (11) $-3 - (-2) + (-3) - 2(-2)$

20. (11) $(-2)(-2)(-2) - |-8|$

21. (11) $-(-5) + (-2) + (-5)|-3|$

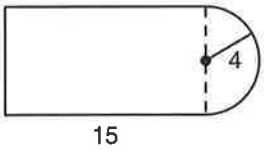
22. (7) $4 + |-3 - 1| + (-3) - (-2)$

23. (11) $3(-2) - |-3 + 6| + 9 - 7(-2)$

24. (10) $\dfrac{-3 + 5}{4 - 6 + 5}$

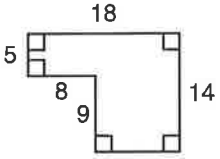
25. (10) $\dfrac{-6 + 4 + 2}{-2 + 5 + 3}$

26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in centimeters.

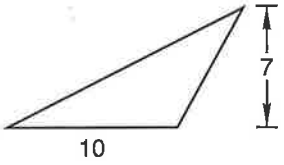


Find the area of each figure. Dimensions are in meters.

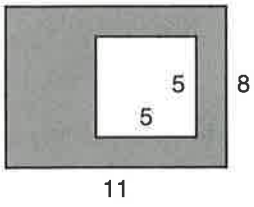
27. (8)



28. (8)



29. (8) Find the area of the shaded portion of this figure. All angles are right angles. Dimensions are in inches.



30. (1) AB is $8\frac{3}{10}$ feet. AC is $26\frac{1}{2}$ feet. Find BC.



LESSON 12 Symbols of Inclusion • Order of Operations

12.A symbols of inclusion

In Lesson 11 we found that the simplification of

$$4 + 3 \cdot 2$$

is 10 because we have agreed to do the multiplication first and then do the addition. So,

$$4 + 3 \cdot 2 = 4 + 6 = 10$$

Parentheses, brackets, braces, and bars are all called **symbols of inclusion,** and can be used to help us emphasize the meaning of our notation. Using these symbols, the notation above could be written in any of the following ways:

(a) $4 + (3 \cdot 2)$ (b) $4 + [3 \cdot 2]$ (c) $4 + \{3 \cdot 2\}$ (d) $4 + \overline{3 \cdot 2}$

Each of the notations emphasizes that 3 is to be multiplied by 2 and that 4 is to be added to this product. A further benefit of the use of symbols of inclusion is that a nonstandard order of operations can be indicated. For example, we can use parentheses to indicate that 4 is to be added to 3 and the result multiplied by 2 by writing

$$2(3 + 4) \quad \text{or} \quad (3 + 4)2$$