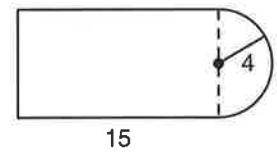
22. (7) $4 + |-3 - 1| + (-3) - (-2)$

23. (11) $3(-2) - |-3 + 6| + 9 - 7(-2)$

24. (10) $\dfrac{-3 + 5}{4 - 6 + 5}$

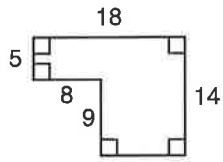
25. (10) $\dfrac{-6 + 4 + 2}{-2 + 5 + 3}$

26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in centimeters.

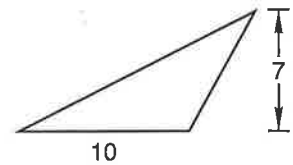


Find the area of each figure. Dimensions are in meters.

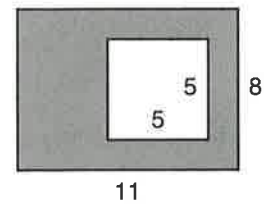
27. (8)



28. (8)



29. (8) Find the area of the shaded portion of this figure. All angles are right angles. Dimensions are in inches.



30. (1) AB is $8\frac{3}{10}$ feet. AC is $26\frac{1}{2}$ feet. Find BC.



LESSON 12 Symbols of Inclusion • Order of Operations

12.A symbols of inclusion

In Lesson 11 we found that the simplification of

$$4 + 3 \cdot 2$$

is 10 because we have agreed to do the multiplication first and then do the addition. So,

$$4 + 3 \cdot 2 = 4 + 6 = 10$$

Parentheses, brackets, braces, and bars are all called **symbols of inclusion,** and can be used to help us emphasize the meaning of our notation. Using these symbols, the notation above could be written in any of the following ways:

(a) $4 + (3 \cdot 2)$ (b) $4 + [3 \cdot 2]$ (c) $4 + \{3 \cdot 2\}$ (d) $4 + \overline{3 \cdot 2}$

Each of the notations emphasizes that 3 is to be multiplied by 2 and that 4 is to be added to this product. A further benefit of the use of symbols of inclusion is that a nonstandard order of operations can be indicated. For example, we can use parentheses to indicate that 4 is to be added to 3 and the result multiplied by 2 by writing

$$2(3 + 4) \quad \text{or} \quad (3 + 4)2$$

While braces and bars can be used as indicated on the previous page, we normally reserve the use of braces to indicate a set, and bars are most often used as fraction lines, as shown here.

$$\frac{4 + (3 \cdot 2)}{5(2 - 3)} = \frac{4 + 6}{5(-1)} = \frac{10}{-5} = -2$$

The parentheses in the numerator are used to emphasize that 4 is to be added to the product of 3 and 2, and the parentheses in the denominator are used to designate that 5 is to be multiplied by the algebraic sum of 2 and –3.

12.B order of operations

To simplify numerical expressions that contain symbols of inclusion, we begin by simplifying within the symbols of inclusion. Then we simplify the resulting expression, remembering that multiplication is performed before addition.

example 12.1 Simplify: $4(3 + 2) - 5(6 - 3)$

solution First we will simplify within the parentheses.

$4(5) - 5(3)$	simplified within parentheses
$20 - 15$	multiplied
5	added algebraically

example 12.2 Simplify: $-3(2 - 3 + 5) - 6(4 + 2) - 3$

solution First we simplify within the parentheses, then multiply, and finish by adding.

$-3(4) - 6(6) - 3$	simplified within parentheses
$-12 - 36 - 3$	multiplied
–51	added algebraically

example 12.3 Simplify: $-2(-3 - 3)(-2 - 4) - (-3 - 2) + 3(4 - 2)$

solution We begin by simplifying within the parentheses.

$-2(-6)(-6) - (-5) + 3(2)$	simplified within parentheses
$-72 + 5 + 6$	multiplied
–61	added algebraically

When the expression is in the form of a fraction, we begin by simplifying both the numerator and the denominator. Then we have our choice of dividing or leaving the result in the form of a fraction.

example 12.4 Simplify: $\dfrac{5(-5 + 3) + 7(-5 + 9) + 2}{(4 - 2) + 3 + 5}$

solution First we will simplify the numerator and the denominator.

$\dfrac{5(-2) + 7(4) + 2}{2 + 3 + 5}$	simplified within parentheses
$\dfrac{-10 + 28 + 2}{2 + 3 + 5}$	multiplied
$\dfrac{20}{10}$	added algebraically
2	divided

example 12.5 Simplify: $\dfrac{-3(4-2)-(-5)}{4-(3)(-3)}$

solution First we will simplify the numerator and the denominator.

$$\frac{-3(2)+5}{4-(3)(-3)}$$ simplified within parentheses

$$\frac{-6+5}{4+9}$$ multiplied

$$\frac{-1}{13}$$ added algebraically

$$-\frac{\mathbf{1}}{\mathbf{13}}$$ this does not divide evenly, and we will leave it in fractional form

practice Simplify:

a. $(-3-2)(-4-1)$

b. $(6-2)-(4-6)$

c. $-5(-3-3)+2(1-3)$

d. $\dfrac{-3(10-8)-(-4)}{4-3(-3)-13}$

problem set 12

1. *(11)* (a) What is the reciprocal of $\frac{1}{2}$?

(b) What is the reciprocal of $-\frac{1}{2}$?

(c) What is the product of any nonzero real number and its reciprocal?

2. *(11)* What is another name for the reciprocal of a number?

3. *(11)* Which real number does not have a reciprocal and why?

4. *(6)* Is the sum of a positive number and a negative number always a negative number?

5. *(2)* (a) What is the degree measure of a right angle?

(b) What is the degree measure of a straight angle?

(c) How many degrees are in a circle?

6. *(4)* Use two unit multipliers to convert 80 feet to centimeters.

7. *(10)* Use two unit multipliers to convert 12 square feet to square inches.

8. *(8)* The area of a rectangle is 18 square centimeters. The length of the rectangle is 6 centimeters. Find the width of the rectangle.

9. *(3)* The circumference of a circle is 6π meters. Find the radius of the circle.

Simplify:

10. *(12)* $(-4+7)+(-3-2)$

11. *(12)* $(-3-2)-(-6+2)$

12. *(12)* $(-2-2)(-3-4)$

13. *(12)* $4(8+4)+7(10-8)$

14. *(12)* $5(9+2)-(-4)(5+1)$

15. *(12)* $-3(-6-2)+3(-2+5)$

16. *(12)* $-2(-5-7)-3(-8+2)$

17. *(12)* $(-3-2)(-2)(-2-2)$

18. *(12)* $(6-2)(-3-5)-(-5)$

19. *(11)* $-8-4-(-2)-(+2)(-3)$

20. *(12)* $\frac{1}{4}(8-4)-5(8-2)-2$

21. *(12)* $5(12+2)-6(-3+8)-(2+3)$

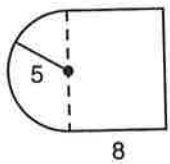
22. (12) $(2 - 3)(-8 + 2) + |-3 + 5|$

23. (12) $-|-2 - 5 + 3|(5 - 2)$

24. (12) $4 - \dfrac{(+12)}{(-3)} + 2$

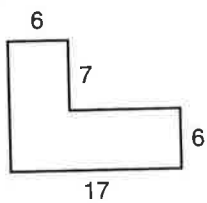
25. (12) $\dfrac{-4(5 - 2) - (-8)}{3 - (-3)(3)}$

26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in kilometers.

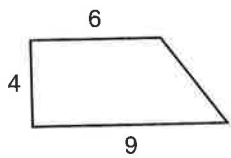


Find the area of each figure. Corners that look square are square. Dimensions are in inches.

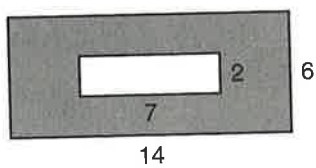
27. (8)



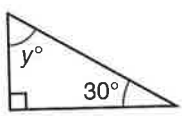
28. (8)



29. (8) Find the area of the shaded portion of this figure. All angles are right angles. Dimensions are in feet.



30. (2) Find y.



LESSON 13 *Multiple Symbols of Inclusion • More on Order of Operations • Products of Signed Numbers*

13.A multiple symbols of inclusion

Often we encounter expressions such as

$$-3[(-2 - 4) - 3] - 2$$

where symbols of inclusion are within other symbols of inclusion. We simplify these expressions by beginning with the innermost symbol of inclusion and working our way out. Here we will simplify within the parentheses and then within the brackets.

$-3[(-6) - 3] - 2$ simplified within parentheses

$-3[-9] - 2$ simplified within brackets

$27 - 2$ multiplied

25 added