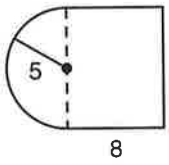
22. (12) $(2 - 3)(-8 + 2) + |-3 + 5|$

23. (12) $-|-2 - 5 + 3|(5 - 2)$

24. (12) $4 - \dfrac{(+12)}{(-3)} + 2$

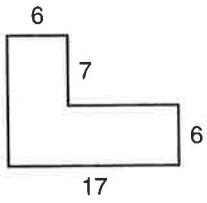
25. (12) $\dfrac{-4(5 - 2) - (-8)}{3 - (-3)(3)}$

26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in kilometers.

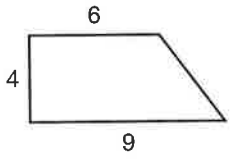


Find the area of each figure. Corners that look square are square. Dimensions are in inches.

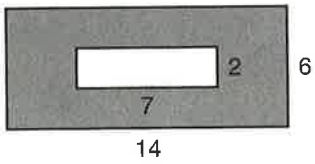
27. (8)



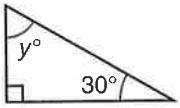
28. (8)



29. (8) Find the area of the shaded portion of this figure. All angles are right angles. Dimensions are in feet.



30. (2) Find y.



LESSON 13 Multiple Symbols of Inclusion • More on Order of Operations • Products of Signed Numbers

13.A multiple symbols of inclusion

Often we encounter expressions such as

$$-3[(-2 - 4) - 3] - 2$$

where symbols of inclusion are within other symbols of inclusion. We simplify these expressions by beginning with the innermost symbol of inclusion and working our way out. Here we will simplify within the parentheses and then within the brackets.

$-3[(-6) - 3] - 2$	simplified within parentheses
$-3[-9] - 2$	simplified within brackets
$27 - 2$	multiplied
25	added

example 13.1 Simplify: $4\{2[(-3 - 2)(-7 + 4) - 5]\} - 2$

solution We will begin on the inside with the parentheses and work our way out.

$4\{2[(-5)(-3) - 5]\} - 2$	simplified within parentheses
$4\{2[10]\} - 2$	simplified within brackets
$4\{20\} - 2$	simplified within braces
$80 - 2$	multiplied
78	added

example 13.2 Simplify: $\dfrac{-3\{[(-2 - 3)][-2]\}}{-3(4 - 2)}$

solution First we will simplify the numerator and the denominator. Then we will divide as the last step.

$\dfrac{-3\{[(-5)][-2]\}}{-3(2)}$	simplified within parentheses
$\dfrac{-3\{10\}}{-3(2)}$	simplified within braces
$\dfrac{-30}{-6}$	multiplied
5	divided

13.B more on order of operations

In the discussion of the order of operations in Lesson 11 we said that mathematicians have agreed that when they write

$$4 \cdot 3 + 2$$

the **multiplication should be done first** and then the addition.

$12 + 2$	multiplied
14	added

We did not discuss division because if symbols of inclusion are properly used, the order in which division is to be performed is apparent.

If we write

$$\frac{4 \cdot 3 + 2}{-7 + 5}$$

we find the value of the numerator and the value of the denominator and then divide.

$$\frac{14}{-2} = -7$$

If the following problem is encountered, however,

$$4 + \frac{14}{2} - 3 \cdot 6$$

the notation clearly indicates that only 14 is to be divided by 2, and if we do this first, we get

$$4 + 7 - 3 \cdot 6$$

Now we do the multiplication and conclude with algebraic addition.

$$4 + 7 - 18$$

$$11 - 18$$

$$-7$$

At this point in an algebra book, however, it is customary to give a rule for finding the number represented by

$$6 + 3 \cdot 6 \div 2 - 6 \cdot 2$$

The rule is to perform the operations from left to right in the following order:

1. Multiplication and division
2. Algebraic addition

First we will go through the problem from **left to right,** performing the multiplications and divisions **in the order in which they are encountered.**

$6 + 3 \cdot 6 \div 2 - 6 \cdot 2$	original problem
$6 + 18 \div 2 - 6 \cdot 2$	multiplied 3 times 6
$6 + 9 - 6 \cdot 2$	divided 18 by 2
$6 + 9 - 12$	multiplied 6 times 2

Now we go through the problem again from left to right, performing the algebraic additions as they are encountered.

$6 + 9 - 12$	from above
$15 - 12$	added 6 and 9
3	added 15 and –12

If symbols of inclusion had been properly used, instead of stating the problem as

$$6 + 3 \cdot 6 \div 2 - 6 \cdot 2$$

the problem would have been written as follows:

$$6 + \frac{(3 \cdot 6)}{2} - (6 \cdot 2)$$

and here the method of solution is clearly indicated. We simplify within the parentheses as the first step.

$6 + \frac{18}{2} - 12$	simplified within parentheses
$6 + 9 - 12$	divided 18 by 2
$15 - 12$	added 6 and 9
3	added 15 and –12

We will use symbols of inclusion to include the use of a bar as a fraction line when stating problems. Thus problems such as the one just discussed will not be encountered again in this book.

13.C products of signed numbers

Let us review the concept of the opposite of a number by watching the pattern that develops here.

	READ AS	WHICH IS
2	2	2
–2	the opposite of 2	–2
–(–2)	the opposite of the opposite of 2	2
–[–(–2)]	the opposite of the opposite of the opposite of 2	–2
–{–[–(–2)]}	the opposite of the opposite of the opposite of the opposite of 2	2

The expressions in the left-hand column are all equivalent expressions for 2 or for –2. If we look at the right-hand column, we see that every time an additional (–) is included in the left-hand expression, the right-hand expression changes sign.

A similar alternation in sign occurs whenever a particular number is multiplied by a negative number. For instance,

$$-2 = -2$$
$$(-2)(-2) = +4$$
$$(-2)(-2)(-2) = -8$$
$$(-2)(-2)(-2)(-2) = +16$$
$$(-2)(-2)(-2)(-2)(-2) = -32$$

The numbers on the right have different absolute values, but they *alternate in sign*. We note that

The product of **two** negative factors is **positive.**

The product of **three** negative factors is **negative.**

The product of **four** negative factors is **positive.**

The product of **five** negative factors is **negative.**

Without proof, we will generalize these observations.

1. **The product of an even number of negative real numbers is a positive real number.**
2. **The product of an odd number of negative real numbers is a negative real number.**

We can use these observations to determine the sign of the product of several signed numbers. Let us consider

$$(4)(-3)(-4)(-2)(11) = ?$$

Here we have +4 and +11 as two of the five factors. Since multiplication by a positive number does not affect the sign of the product, we will not consider these numbers. The other three factors are negative. We can look at the rules stated previously and see that the sign of the product of three negative numbers is negative. Thus our answer can be expressed as

$$-(4 \cdot 3 \cdot 4 \cdot 2 \cdot 11) = -1056$$

example 13.3 Determine the signs of the following products and give the reasons. Do not do the multiplications.

(a) $(-4)(-3)(2)(+5)(+6)$

(b) $(3)(+2)(6)$

(c) $(-3)(-2)(6)(4)(-2)$

(d) $(-3)(-2)(-5)(-7)(-2)$

(e) $(-3)(-4)(-2) + 2(-3)$

solution

		SIGN	REASON
(a)	$(-4)(-3)(2)(+5)(+6)$	**positive**	Even number of negative factors
(b)	$(3)(+2)(6)$	**positive**	No negative factors
(c)	$(-3)(-2)(6)(4)(-2)$	**negative**	Odd number of negative factors
(d)	$(-3)(-2)(-5)(-7)(-2)$	**negative**	Odd number of negative factors
(e)	$(-3)(-4)(-2) + 2(-3)$	?	Rule does not apply, as this is an indicated *sum*. We will do the problem in three steps.
	1. $(-3)(-4)(-2)$	negative	Odd number of negative factors
	2. $2(-3)$	negative	Odd number of negative factors
	3. $-(3)(4)(2) - (2)(3)$	**negative**	Algebraic sum of two negative numbers is a negative number

practice Simplify:

a. $3\{2[(-4 - 3)(-8 - 2) - 4]\}$

b. $\dfrac{-3\{[(-4 - 1)3] - 5\}}{2(4 - 7)}$

c. $(-2)(-2)(-3)(-3)$

d. $-\{-[-(-2)]\}$

e. Is the following product a positive number or negative number? Do not multiply. Give the reason.

$$4(-3)(5)(-4)(-3)(-7)(-21)(5)(14)(-5)(-8)$$

problem set 13

1. *(13)* Is the product of 5 negative numbers and 2 positive numbers a positive number or a negative number?

2. *(11)* (a) What is the multiplicative inverse of 3?

(b) What is the multiplicative inverse of –3?

(c) What is the product of any nonzero real number and its multiplicative inverse?

3. *(11)* What is another name for the multiplicative inverse of a number?

4. *(11)* Which real number does not have a multiplicative inverse and why?

5. *(2)* What do you call polygons in which all sides have the same length?

6. *(4)* Use two unit multipliers to convert 60 miles to inches.

7. *(10)* Use two unit multipliers to convert 125 square miles to square feet.

8. *(8)* The area of a rectangle is 35 square centimeters. The width of the rectangle is 5 centimeters. Find the length of the rectangle.

9. (8) The diameter of a circle is 6 meters. Find the area of the circle.

Simplify:

10. (7,13) $-[-(-2)]$

11. (7,13) $-\{-[-(-3)]\}$

12. (12) $(-3 - 2) - (5 + 2)$

13. (12) $(-3 + 5)(2 - 3)$

14. (12) $-2(-6 - 3) + \frac{0}{5}$

15. (11) $-2 + (-2) - (-4)5$

16. (11) $-3 - (2) + (-2) - (-3)(-2)$

17. (12) $-5(-3 - 2) + (-2) - (-3 - 4)$

18. (12) $(-2)(-3)(-4 + 2) - (3 + 1)$

19. (12) $-2 - (-2) - |-2|(2)$

20. (13) $\frac{-6}{-10} + (-3)(-2) + 3|-4 - 2|$

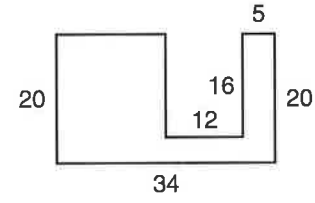
21. (13) $(-7) - [-(-2)]5$

22. (13) $-3 - [-(-2)] + (-3)(5)$

23. (12) $\frac{-2(-6) - 2}{-3 + (-7 + 2)}$

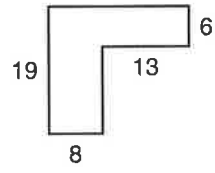
24. (10) $\frac{4 + 2 - 3(2)}{3(2) - 6}$

25. (3) Find the perimeter of this figure. All angles are right angles. Dimensions are in inches.

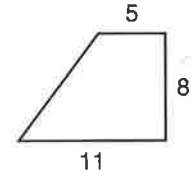


Find the area of each figure. Corners that look square are square. Dimensions are in feet.

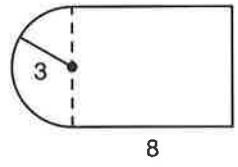
26. (8)



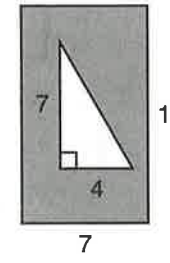
27. (8)



28. (8)



29. (8) Find the area of the shaded portion of this rectangle. Dimensions are in yards.



30. (1) EF is $4\frac{2}{15}$ miles. DF is $15\frac{4}{5}$ miles. Find DE.

