LESSON 14 Evaluation of Algebraic Expressions

In Lesson 4, we said that a **number** is an **idea** and that when we wish to write down something to represent this **idea,** we use a **numeral.** If we wish to bring to mind the number 7, we could write any of the following:

$$7 \qquad \frac{14}{2} \qquad 4 + 3 \qquad \frac{-21}{-3} \qquad 2 + 2 + 2 + 1$$

We call each of these notations a **numerical expression** or just a **numeral.** Every numerical expression represents only one number and we call this number the **value** of the expression. Each of the numerical expressions shown above has a **value** of 7.

In algebra we often use letters to represent numbers. When letters as well as numbers are used in an expression, we do not call the expression a numerical expression but we call it by the more general name of **algebraic expression** or **mathematical expression.** These words are used to describe expressions that contain only numbers or only letters or contain both numbers and letters.

If we write the algebraic expression

$$4 + x$$

the algebraic expression has a **value** that depends on the value that we assign to x. If we give x a value of 5, then the algebraic expression has a value of 9 because

$$4 + 5 = 9$$

If we give x a value of 11, then the algebraic expression has a value of 15 because

$$4 + 11 = 15$$

Because the value assigned to x can be changed or varied, we call letters such as x **variables.** We also call the letters **unknowns** since they represent unknown or unspecified numbers. The numeral 4 in this example does not change value and has a constant value of 4. For this reason the symbol that we use to denote a number is called a **constant.**

When we use variables in algebraic expressions, the notations that we use to indicate the operations of division and algebraic addition are the same as the notations that we use to indicate the division and algebraic addition of real numbers. The notation for the multiplication of variables is sometimes slightly different. We can denote that we wish to multiply 4 by the variable x by writing any of the following:

(a) $4x$ (b) $4(x)$ (c) $(4)(x)$ (d) $4 \cdot x$ (e) $(4) \cdot (x)$

The notations (b) through (e) are the same as the notations that we use for real number multiplication, but the notation shown in (a) is different from the real number notation.

$4x$ indicates that 4 is to be multiplied by the value of x

whereas

45 does not indicate that 4 is to be multiplied by 5, but instead is a numeral that represents the number 45

Thus the algebraic expression xym indicates that the values of x, y, and m are to be multiplied. If we give x a value of 1, y a value of 2, and m a value of 3, the value of the algebraic expression xym can be found.

$$1 \cdot 2 \cdot 3 = 6$$

If we write the algebraic expression

$$4x + mx$$

we indicate that 4 is to be multiplied by the value of x and that the value of m is to be multiplied by the value of x and that the two products are to be added. If we give x a value of 3 and m a value of 5, then we can find the value of the algebraic expression.

$$4 \cdot 3 + 5 \cdot 3$$

$$12 + 15 \quad = 27$$

Thus the value of the algebraic expression when x equals 3 and m equals 5 is 27.

If we give x the value of 2 and m the value of 6, then the algebraic expression will have a different value.

$$4x + mx = 4 \cdot 2 + 6 \cdot 2 = 8 + 12 = 20$$

In this case, the value of the algebraic expression is 20.

It is of **utmost importance** to note that in the first case, when we gave x a value of 3, the value of x everywhere in the algebraic expression was 3. When we gave x a value of 2, the value of x everywhere in the algebraic expression was 2. **While the values assigned to variables may change or be changed, under any set of conditions the value assigned to a particular variable in an algebraic expression is the same value throughout the algebraic expression. Also, when we begin solving equations and working problems, we must remember that the value assigned to any particular variable under any set of conditions must be the same value regardless of where the particular variable appears in the equation or the problem.**

example 14.1 Find the value of: xmp if $x = 4$, $m = 5$, and $p = 2$

solution We replace x with 4, m with 5, and p with 2.

$$xmp = 4 \cdot 5 \cdot 2 = 20 \cdot 2 = \mathbf{40}$$

example 14.2 Evaluate (find the value of): $4yz - 5$ if $y = 2$ and $z = 10$

solution We replace y with 2 and z with 10 and then simplify.

$$4yz - 5 = 4(2)(10) - 5 = 80 - 5 = \mathbf{75}$$

example 14.3 Evaluate: $y - z$ if $y = -2$ and $z = -6$

solution We replace y with -2 and $-z$ with $+6$ since $-z$ represents the opposite of z.

$$y - z = -2 + 6 = \mathbf{+4}$$

example 14.4 Evaluate: $-a - b - ab$ if $a = -3$ and $b = -4$

solution The value of a is -3, so the opposite of a is 3. The value of b is -4, so the opposite of b is 4. Finally, $ab = 12$ and the opposite of this is -12. Thus, we get -5 for an answer.

$$3 + 4 - 12 = \mathbf{-5}$$

example 14.5 Evaluate: $-x - (-a + b)$ if $x = 2$, $a = -4$, and $b = -6$

solution Some people find that it is helpful to replace each variable with parentheses. Then the proper number is written inside the parentheses.

$$-(\ ) - [-(\ ) + (\ )] \quad \text{replaced variables with parentheses}$$

$$-(2) - [-(-4) + (-6)] \quad \text{numbers inserted}$$

The first entry can be read as the opposite of 2, or –2. Inside the brackets we have –(–4), read "the opposite of the opposite of 4," which is 4 itself. The last entry inside the brackets is +(–6), read "plus the opposite of 6," which is the same as –6. Thus we have

$$-2 - (4 - 6) = -2 - (-2) = -2 + 2 = \mathbf{0}$$

example 14.6 Evaluate: $x - y(-a + x)$ if $x = -2$, $y = +3$, and $a = -4$

solution We will replace each variable with parentheses.

$(\ ) - (\ )[-(\ ) + (\ )]$	replaced variables with parentheses
$(-2) - (+3)[-(-4) + (-2)]$	numbers inserted
$= -2 - (3)(4 - 2) = -2 - (3)(2)$	simplified
$= -2 - 6 = \mathbf{-8}$	simplified

example 14.7 Evaluate: $-(m + x)(-a + mx)$ if $m = 2$, $x = -3$, and $a = -4$

solution We will replace each variable with parentheses.

$-[(\ ) + (\ )][-(\ ) + (\ )(\ )]$	replaced variables with parentheses
$-[(2) + (-3)][-(-4) + (2)(-3)]$	numbers inserted
$= -(2 - 3)[4 + (-6)] = -(-1)(4 - 6)$	simplified
$= -(-1)(-2) = -(2) = \mathbf{-2}$	simplified

example 14.8 Evaluate: $-xa(x - a) + a$ if $a = -2$ and $x = 4$

solution We will replace each variable with parentheses.

$-(\ )(\ )[(\ ) - (\ )] + (\ )$	replaced variables with parentheses
$-(4)(-2)[(4) - (-2)] + (-2)$	numbers inserted
$= -(-8)(4 + 2) - 2 = 8(6) - 2$	simplified
$= 48 - 2 = \mathbf{46}$	simplified

practice Evaluate:

a. $x - xy$ if $x = -2$ and $y = 3$

b. $a - (ab - a)$ if $a = -4$ and $b = -2$

c. $x - ab(a - b)$ if $x = -3$, $a = -2$, and $b = -4$

d. $-xa(a + x) + x$ if $x = -4$ and $a = -2$

problem set 14

1. *(14)* What is the difference between a numerical expression and an algebraic expression?

2. *(14)* What do we mean by the value of an expression?

3. *(14)* (a) What is a variable of an algebraic expression?

(b) What is an unknown of an algebraic expression?

4. *(13)* Is the product of 6 negative numbers and 5 positive numbers a positive number or a negative number?

5. (2) What do you call polygons in which all angles have the same measure?

6. (4) Use two unit multipliers to convert 300 inches to meters.

7. (10) Use two unit multipliers to convert 100 square yards to square feet.

8. (8) The area of a rectangle is 44 square inches. The length of one side is 11 inches. What is the length of the other side?

9. (3) The circumference of a circle is 8π feet. Find the radius of the circle.

Evaluate:

10. (14) $xm - 2m$ if $x = -2$ and $m = -3$

11. (14) $ma - m - a$ if $m = -2$ and $a = -4$

12. (14) $2abc - 3ab$ if $a = 2$, $b = -3$, and $c = 4$

13. (14) $-x(a + b)$ if $x = 4$, $a = -3$, and $b = -5$

14. (14) $-a + b + ab$ if $a = -5$ and $b = -2$

15. (14) $x - y(a - x)$ if $x = -3$, $y = 4$, and $a = 4$

16. (14) $-(m - x)(a - mx)$ if $m = 3$, $x = -4$, and $a = -2$

17. (14) $-xa(x + a) - a$ if $a = -4$ and $x = 2$

18. (14) $-xy - (-x + y)$ if $x = -3$ and $y = -4$

Simplify:

19. (12) $-2(-1 - 4)(5 - 6) + 3$

20. (13) $-4 - [-(-5)] + |-6|$

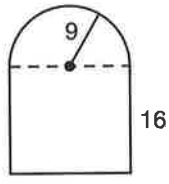
21. (13) $-|-3|(2 - 5) - [-(-3)]$

22. (13) $-2[-3(-2 - 5)(3)]$

23. (13) $\dfrac{-2[-(-3)]}{(-2)(-4 + 3)}$

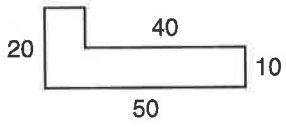
24. (12) $\dfrac{(-5 - 2) + (-3 - 2)}{-3 - (-2)}$

25. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in yards.

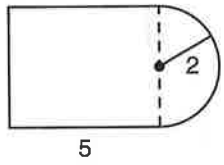


Find the area of each figure. Corners that look square are square. Dimensions are in centimeters.

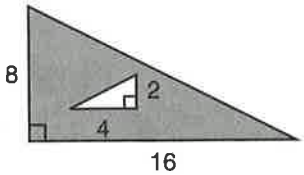
26. (8)



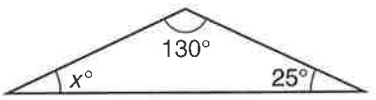
27. (8)



28. (8) Find the area of the shaded portion of this figure. Dimensions are in meters.



29. (2) Find x.



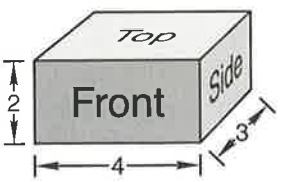
30. (4) Simplify: $\dfrac{0.0612}{1.02}$

LESSON 15 *Surface Area*

Geometric figures that have three dimensions are called **geometric solids.** Geometric solids whose sides are at right angles to the bases are called **right geometric solids** or just **right solids.** The "top" and the "bottom" of a right solid are called the **bases** of the right solid and are identical geometric figures. If the bases of a right solid are polygons, the right solid is called a **right prism.**

The **surface area** of a geometric solid is the total area of all the exposed surfaces of the geometric solid. In this lesson, we will learn how to compute the surface area of right solids.

example 15.1 Find the surface area of this right rectangular prism. Dimensions are in centimeters.



solution A right rectangular prism has six faces that are rectangles. It has a top and a bottom, a front and a back, and two sides. The areas of the top and the bottom are equal. The area of the front equals the area of the back, and the areas of the two sides are equal. The surface area of the right rectangular prism is equal to the sum of the areas of the six faces.

$$\begin{aligned}
\text{Area of top} &= 4\text{ cm} \times 3\text{ cm} = 12\text{ cm}^2 \\
\text{Area of bottom} &= 4\text{ cm} \times 3\text{ cm} = 12\text{ cm}^2 \\
\text{Area of front} &= 4\text{ cm} \times 2\text{ cm} = 8\text{ cm}^2 \\
\text{Area of back} &= 4\text{ cm} \times 2\text{ cm} = 8\text{ cm}^2 \\
\text{Area of side} &= 3\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2 \\
\text{Area of side} &= 3\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2 \\
\text{Surface area} &= \text{Total} = \mathbf{52\text{ cm}^2}
\end{aligned}$$

example 15.2 Find the surface area of this right triangular prism. Dimensions are in meters.

