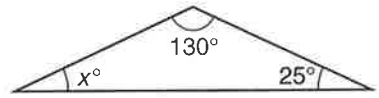
29. (2) Find x.



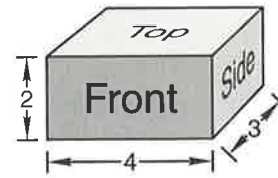
30. (4) Simplify: $\dfrac{0.0612}{1.02}$

LESSON 15 *Surface Area*

Geometric figures that have three dimensions are called **geometric solids.** Geometric solids whose sides are at right angles to the bases are called **right geometric solids** or just **right solids.** The "top" and the "bottom" of a right solid are called the **bases** of the right solid and are identical geometric figures. If the bases of a right solid are polygons, the right solid is called a **right prism.**

The **surface area** of a geometric solid is the total area of all the exposed surfaces of the geometric solid. In this lesson, we will learn how to compute the surface area of right solids.

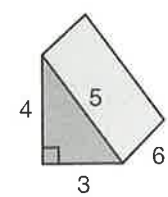
example 15.1 Find the surface area of this right rectangular prism. Dimensions are in centimeters.



solution A right rectangular prism has six faces that are rectangles. It has a top and a bottom, a front and a back, and two sides. The areas of the top and the bottom are equal. The area of the front equals the area of the back, and the areas of the two sides are equal. The surface area of the right rectangular prism is equal to the sum of the areas of the six faces.

$$\begin{aligned}
\text{Area of top} &= 4\text{ cm} \times 3\text{ cm} = 12\text{ cm}^2 \\
\text{Area of bottom} &= 4\text{ cm} \times 3\text{ cm} = 12\text{ cm}^2 \\
\text{Area of front} &= 4\text{ cm} \times 2\text{ cm} = 8\text{ cm}^2 \\
\text{Area of back} &= 4\text{ cm} \times 2\text{ cm} = 8\text{ cm}^2 \\
\text{Area of side} &= 3\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2 \\
\text{Area of side} &= 3\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2 \\
\text{Surface area} &= \text{Total} = \mathbf{52\text{ cm}^2}
\end{aligned}$$

example 15.2 Find the surface area of this right triangular prism. Dimensions are in meters.



solution A right triangular prism has five faces. It has two faces that are right triangles and three faces that are rectangles. The surface area of the right triangular prism is equal to the sum of the areas of the five faces.

$$\text{Area of one end} = \frac{3\text{ m} \times 4\text{ m}}{2} = 6\text{ m}^2$$

$$\text{Area of one end} = \frac{3\text{ m} \times 4\text{ m}}{2} = 6\text{ m}^2$$

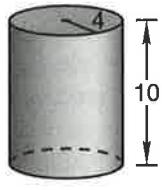
$$\text{Area of bottom} = 3\text{ m} \times 6\text{ m} = 18\text{ m}^2$$

$$\text{Area of back} = 4\text{ m} \times 6\text{ m} = 24\text{ m}^2$$

$$\text{Area of front} = 5\text{ m} \times 6\text{ m} = 30\text{ m}^2$$

$$\text{Surface area} = \text{Total} = \mathbf{84\text{ m}^2}$$

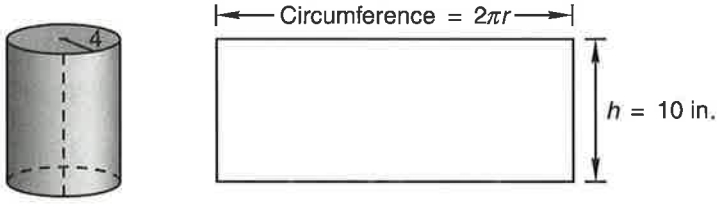
example 15.3 A right circular cylinder has a radius of 4 inches and a height of 10 inches, as shown. Find the surface area of the right circular cylinder.



solution The right circular cylinder has two equal bases that are circles. So the area of a base is the area of a circle.

$$\text{Area of one base} = \pi r^2 = (3.14)(4\text{ in.})^2 = 50.24\text{ in.}^2$$

We define the **lateral surface area** of a right circular cylinder to be the area of the curved surface between the bases. **The lateral surface area of a right circular cylinder is equal to the perimeter of a base times the height.** We can see this if we think of the right circular cylinder as a tin can that we can cut down the dotted line and then press flat. When we do this we get a rectangle.



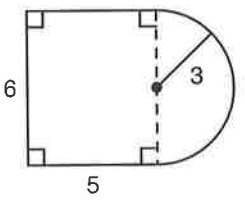
The length of the rectangle is the perimeter of a base of the right circular cylinder. We note that the perimeter of a base is the circumference of a circle. The height of the rectangle is the height of the right circular cylinder. The lateral surface area of the right circular cylinder is the area of the rectangle which is equal to the perimeter of a base times the height. Therefore, we have

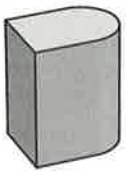
$$\begin{aligned}\text{Lateral surface area} &= (\text{perimeter of a base})(\text{height}) \\ &= (\text{circumference})(\text{height}) \\ &= 2\pi rh \\ &= 2(3.14)(4\text{ in.})(10\text{ in.}) \\ &= 251.2\text{ in.}^2\end{aligned}$$

The surface area of a right circular cylinder is equal to the areas of the two equal bases added to the lateral surface area.

$$\begin{aligned}\text{Surface area} &= \text{base area} + \text{base area} + \text{lateral surface area}\\ &= 50.24 \text{ in.}^2 + 50.24 \text{ in.}^2 + 251.2 \text{ in.}^2\\ &= \mathbf{351.68 \text{ in.}^2}\end{aligned}$$

example 15.4 A base of the right solid 10 feet high is shown. Find the surface area of the right solid. Dimensions are in feet.

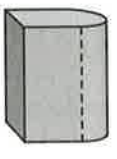


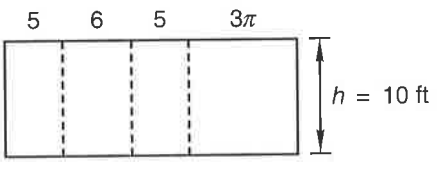


solution The right solid has two equal bases. The area of a base equals the sum of the areas of the rectangle and the semicircle.

$$\begin{aligned}\text{Area of one base} &= \square + \text{D}\\ &= (5 \text{ ft} \times 6 \text{ ft}) + \frac{\pi(3 \text{ ft})^2}{2}\\ &= 30 \text{ ft}^2 + \frac{(3.14)(3 \text{ ft})^2}{2}\\ &= 30 \text{ ft}^2 + 14.13 \text{ ft}^2\\ &= 44.13 \text{ ft}^2\end{aligned}$$

We define the **lateral surface area** of a right solid to be the area of the surface between the bases. **The lateral surface area of a right solid is equal to the perimeter of a base times the height.** We can see this if we cut the right solid down the dotted line and then press flat. When we do this we get a rectangle.





The length of the rectangle is the perimeter of a base of the right solid. The perimeter of a base is equal to the sum of the lengths of the straight sides and the length of the semicircle.

$$\begin{aligned}\text{Perimeter of a base} &= 5 \text{ ft} + 6 \text{ ft} + 5 \text{ ft} + \frac{2\pi(3 \text{ ft})}{2}\\ &= 5 \text{ ft} + 6 \text{ ft} + 5 \text{ ft} + 3\pi \text{ ft}\\ &= 16 \text{ ft} + 3(3.14) \text{ ft}\\ &= 25.42 \text{ ft}\end{aligned}$$

The height of the rectangle is the height of the right solid. The lateral surface area of the right solid is the area of the rectangle which is equal to the perimeter of a base times the height. Therefore, we have

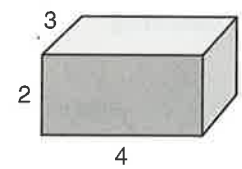
$$\begin{aligned}\text{Lateral surface area} &= (\text{perimeter of a base})(\text{height}) \\ &= (25.42 \text{ ft})(10 \text{ ft}) \\ &= 254.2 \text{ ft}^2\end{aligned}$$

The surface area of a right solid is equal to the areas of the two equal bases added to the lateral surface area.

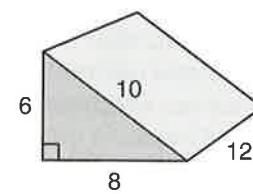
$$\begin{aligned}\text{Surface area} &= \text{base area} + \text{base area} + \text{lateral surface area} \\ &= 44.13 \text{ ft}^2 + 44.13 \text{ ft}^2 + 254.2 \text{ ft}^2 \\ &= \mathbf{342.46 \text{ ft}^2}\end{aligned}$$

practice

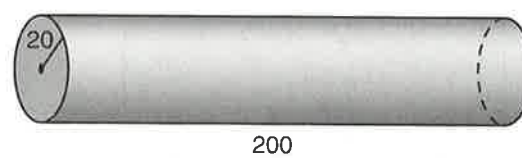
a. Find the surface area of this right rectangular prism. Dimensions are in inches.



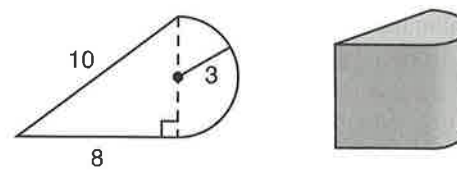
b. Find the surface area of this right triangular prism. Dimensions are in feet.



c. A right circular cylinder has a radius of 20 centimeters and a length of 200 centimeters, as shown. Find the surface area of the right circular cylinder.



d. A base of the right solid 10 meters high is shown. Find the surface area of the right solid. Dimensions are in meters. (Remember that the lateral surface area of a right solid is equal to the perimeter of a base times the height.)



problem set 15

1. *(15)* (a) What do we call the total area of all exposed surfaces of a geometric solid?

(b) How do you find the lateral surface area of any right solid?

2. (11) (a) What is the multiplicative inverse of $\frac{1}{3}$?

(b) What is the multiplicative inverse of $-\frac{1}{3}$?

(c) What is the product of any nonzero real number and its multiplicative inverse?

3. (11) What is another name for the multiplicative inverse of a number?

4. (11) Which real number does not have a multiplicative inverse and why?

5. (4) Use two unit multipliers to convert 112 feet to centimeters.

6. (10) Use two unit multipliers to convert 60 square kilometers to square meters.

7. (8) The area of a rectangle is 72 square centimeters. The length of one side is 6 centimeters. What is the length of the other side?

8. (8) The length of the base of a right triangle is 12 meters. The height of the right triangle is 9 meters. Find the area of the right triangle.

9. (8) The diameter of a circle is 16 kilometers. Find the area of the circle.

Evaluate:

10. (14) $a - ab$ if $a = 2$ and $b = -3$

11. (14) $xy - 3y$ if $x = 2$ and $y = 4$

12. (14) $2ab - 3abc$ if $a = -1$, $b = 2$, and $c = 3$

13. (14) $-x(a - b)$ if $x = -2$, $a = 3$, and $b = -1$

14. (14) $-x - (-a + b)$ if $x = 3$, $a = -2$, and $b = 4$

15. (14) $(x - y)(y - x)$ if $x = -2$ and $y = 3$

16. (14) $(-x) + (-y)$ if $x = -3$ and $y = 2$

17. (14) $-c - (p - c)$ if $p = -5$ and $c = 2$

18. (14) $(a - x)(ma - x)$ if $a = -3$, $x = -4$, and $m = 5$

Simplify:

19. (12) $-2(-6 - 1 - 2) - (-2 + 7)$

20. (12) $-|-11| + (-3)|-3 + 5|$

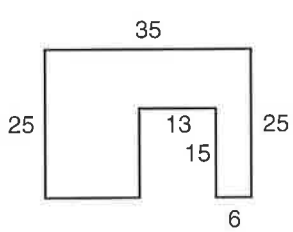
21. (12) $-3(-3)(-2 - 5 + |-11|)$

22. (13) $-3\{[(-5 - 2)](-1)\}$

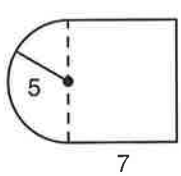
23. (10) $\dfrac{-4 - (-1 - 3)}{-7 - (-9 + 2)}$

24. (12) $\dfrac{3(-5 + 2) + 6(-4 + 10) - 2}{(10 - 2) + 7 + 5}$

25. (3) Find the perimeter of this figure. All angles are right angles. Dimensions are in inches.



26. (8) Find the area of this figure. Corners that look square are square. Dimensions are in feet.



Simplify:

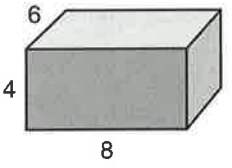
27. (1) $-2\frac{2}{3} + 2\frac{3}{5}$

28. (4) $\dfrac{4\frac{2}{3}}{-3\frac{1}{9}}$

29. (1) The length of $\overline{PQ}$ is $5\frac{1}{3}$ yards. The length of $\overline{QR}$ is $8\frac{5}{12}$ yards. Find PR.



30. (15) Find the surface area of this right rectangular prism. Dimensions are in centimeters.



LESSON 16 More Complicated Evaluations

The procedures discussed in Lesson 14 are also used to evaluate more complicated expressions. The use of parentheses, brackets, and braces is often helpful in preventing mistakes. We will use all of these symbols of inclusion in the following examples.

example 16.1 Evaluate: $-a[-a(p - a)]$ if $p = -2$ and $a = -4$

solution We use parentheses, brackets, and braces as required.

$$-(\ )\{-(\ )[(\ ) - (\ )]\}$$

Now we will insert the numbers inside the parentheses.

$$-(-4)\{-(-4)[(-2) - (-4)]\}$$

Lastly, we simplify.

$$4\{4[2]\} = 4\{8\} = \mathbf{32}$$

example 16.2 Evaluate: $ax[-a(a - x)]$ if $a = -2$ and $x = -6$

solution This time we will not use parentheses. We will replace a with -2, $-a$ with 2, x with -6, and $-x$ with 6.

$$12[2(-2 + 6)]$$

Now we simplify, remembering to begin with the innermost symbol of inclusion.

$$12[2(-2 + 6)] = 12[2(4)] = 12[8] = \mathbf{96}$$

example 16.3 Evaluate: $-b[-b(b - c) - (c - b)]$ if $b = -4$ and $c = -6$

solution We replace b with -4, $-b$ with 4, c with -6, and $-c$ with 6.

$$4[4(-4 + 6) - (-6 + 4)]$$

Now we simplify, remembering to begin within the innermost symbols of inclusion and to multiply before adding.

$$4[4(2) - (-2)] = 4[8 + 2] = 4[10] = \mathbf{40}$$