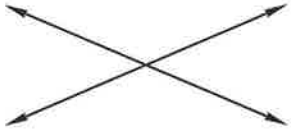
LESSON 2 Angles • Polygons • Triangles • Quadrilaterals

2.A angles

If two lines cross, we say that the lines **intersect.** The place where the lines cross is called the **point of intersection.** Two lines in the same plane either intersect or do not intersect. If two lines in the same plane do not intersect, we say that the lines are **parallel lines.** The perpendicular distance between two parallel lines is everywhere the same.

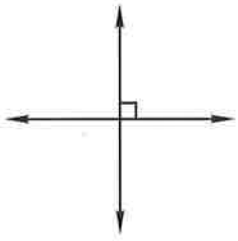


Intersecting lines



Parallel lines

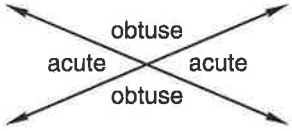
If two lines make square corners at the point of intersection, we say that the lines are **perpendicular.** The angles made by perpendicular lines are called **right angles.** We can draw a little square at the point of intersection to indicate that all four angles formed are right angles. Two right angles form a **straight angle.** An angle smaller than a right angle is called an **acute angle.** An angle greater than a right angle but less than a straight angle is called an **obtuse angle.**



4 right angles

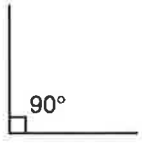


Straight angle

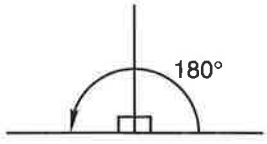


2 acute angles and 2 obtuse angles

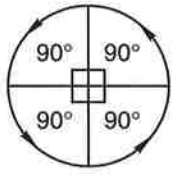
If a right angle is divided into 90 parts, we say that each part has a measure of 1 degree. Thus, a right angle has a measure of 90 degrees, which can also be written as 90°. Two right angles form a straight angle. Thus, a straight angle has a measure of 180 degrees, which can also be written as 180°. Four right angles have a measure of 360 degrees, which can also be written as 360°. Thus, the measure of a circle is 360°.



90° in a right angle



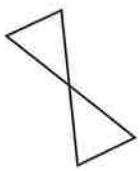
180° in a straight angle



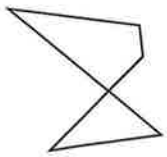
360° in a circle

2.B polygons

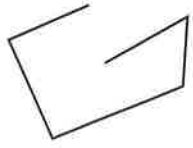
A **polygon** is a special kind of geometric figure. The word *polygon* is formed from the Greek roots *poly*, which means "more than one" or "many," and *gonon*, which means "angle." Thus, polygon literally means "more than one angle." Modern authors define polygons as simple, closed, flat geometric figures whose sides are line segments. The following are examples of figures that are not polygons.



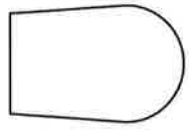
(a) Not a polygon



(b) Not a polygon



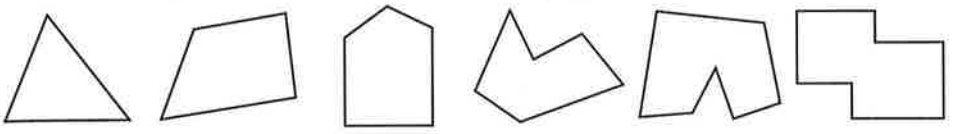
(c) Not a polygon



(d) Not a polygon

Figures (a) and (b) are not polygons because the line segments cross and therefore the figures are not simple figures. Figure (c) is not a polygon because it is not closed. Figure (d) is not a polygon because one of the "sides" is curved.

The following are examples of figures that are polygons:



Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
3 sides	4 sides	5 sides	6 sides	7 sides	8 sides
3 vertices	4 vertices	5 vertices	6 vertices	7 vertices	8 vertices

Each segment of a polygon is called a **side.** Each endpoint of a side is called a **vertex** of the polygon. The plural of vertex is **vertices. For each polygon, the number of sides is always equal to the number of vertices.**

Polygons are named according to the number of sides they have:

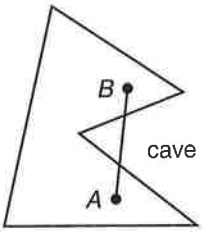
- The polygon with the fewest number of sides is the triangle.
- A polygon with 4 sides is called a *quadrilateral.*
- A polygon with 5 sides is called a *pentagon.*
- A polygon with 6 sides is called a *hexagon.*
- A polygon with 7 sides is called a *heptagon.*
- A polygon with 8 sides is called an *octagon.*
- A polygon with 9 sides is called a *nonagon.*
- A polygon with 10 sides is called a *decagon.*
- A polygon with 11 sides is called an *undecagon.*
- A polygon with 12 sides is called a *dodecagon.*

Some polygons of more than 12 sides have special names, but these names are not used often. Instead, we use the word *polygon* and tell the number of sides or use the number of sides with the suffix *-gon.* Thus, if a polygon has 42 sides, we would call it a polygon with 42 sides or a 42-gon. For easy reference, we list the names of special polygons in the following table.

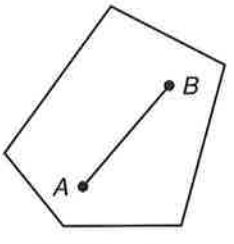
Number of Sides	Name of Polygon	Number of Sides	Name of Polygon
3	Triangle	9	Nonagon
4	Quadrilateral	10	Decagon
5	Pentagon	11	Undecagon
6	Hexagon	12	Dodecagon
7	Heptagon	n	n-gon
8	Octagon		

concave and convex polygons

If a polygon has an indentation (a cave), the polygon is called a **concave polygon.** Any polygon that does not have an indentation is called a **convex polygon.** Any two points in the interior of a convex polygon can be connected by a line segment that does not cut a side of the polygon. This statement is not true for concave polygons. Most of the polygons that you will study will be convex polygons. In this book, when *polygon* is used it will mean *convex polygon* unless stated otherwise.



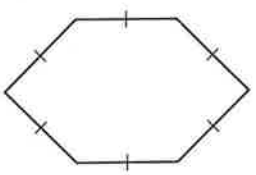
Concave polygon



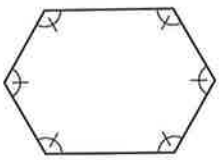
Convex polygon

regular polygons

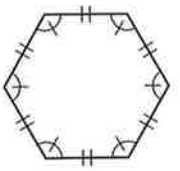
If all the sides of a polygon have the same length, the polygon is called an **equilateral polygon.** If all the angles of a polygon have the same measure, the polygon is called an **equiangular polygon.** Polygons in which all sides have the same length and all angles have the same measure are called **regular polygons.** In some of the following figures we use **tick marks** to denote sides whose lengths are equal and angles whose measures are equal.



Equilateral polygon

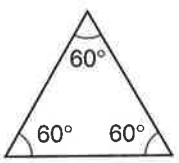


Equiangular polygon

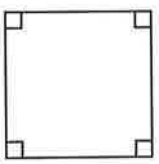


Regular polygon

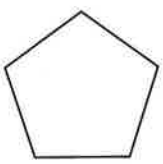
The following are examples of polygons that are regular polygons.



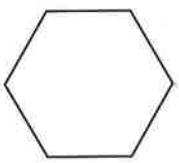
Regular triangle (Equilateral triangle)



Regular quadrilateral (Square)



Regular pentagon



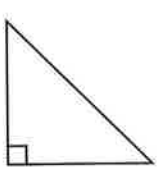
Regular hexagon

A regular triangle is called an equilateral triangle. All the angles in an equilateral triangle are 60° angles. We remember that a quadrilateral is a polygon with four sides. A regular quadrilateral is a square.

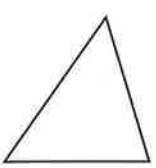
2.C triangles

We remember that the polygon with the fewest number of sides is the triangle. Triangles have three sides and three angles. **The sum of the measures of the three angles in any triangle is 180°.** Triangles can be classified according to the measures of their angles or according to the lengths of their sides.

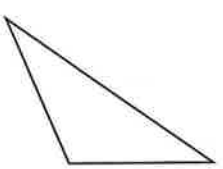
If a triangle has a right angle, the triangle is a **right triangle.** If all angles have a measure less than 90°, the triangle is an **acute triangle.** If one angle has a measure greater than 90°, the triangle is an **obtuse triangle.** The Latin prefix *equi-* means "equal" and the Latin word *angulus* means "angle." We can put them together to form *equiangular*, which means "equal angles." An **equiangular triangle** is a triangle in which the measures of all angles are equal. Each angle in an equiangular triangle must have a measure of 60° because $3 \times 60°$ equals 180°.



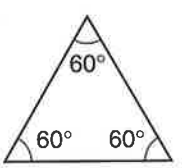
Right triangle



Acute triangle



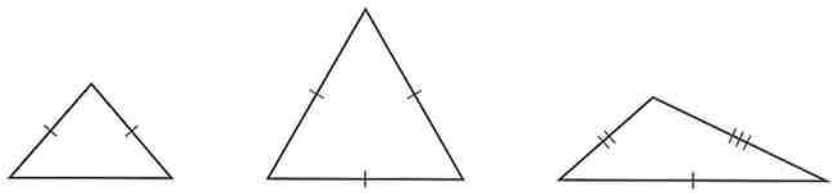
Obtuse triangle



Equiangular triangle

Triangles are also classified according to the relative lengths of their sides. The Greek prefix *iso-* means "equal" and the Greek word *skelos* means "leg." We can put them together to form *isosceles*, which means "equal legs." An **isosceles triangle** is a triangle that has at least two sides of equal length. Since the Latin prefix *equi-* means "equal" and the Latin word *latus* means "side," we can put them together to form *equilateral*, which means "equal sides." An

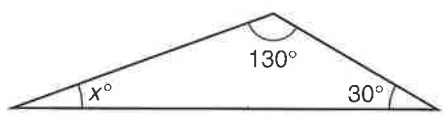
equilateral triangle is a triangle in which the lengths of all sides are equal. If all the sides of a triangle have different lengths, the triangle is called a **scalene triangle.**



Isosceles triangle Equilateral triangle Scalene triangle

The lengths of the sides of a triangle and the measures of the angles opposite these sides are related. In any triangle, the angles opposite sides of equal lengths have equal measures. Also, the sides opposite angles of equal measures have equal lengths. We remember an isosceles triangle is a triangle that has at least two sides of equal length. The angles opposite these sides have equal measures. All three sides in an equilateral triangle have the same length. Since the angles opposite these sides have equal measures, an equilateral triangle is also an equiangular triangle. All three angles in an equiangular triangle have equal measures. Since the sides opposite these angles have equal lengths, an equiangular triangle is also an equilateral triangle. The scalene triangle has no equal sides, so no two angles have equal measures.

example 2.1 Find x.

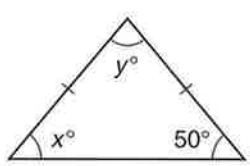


solution **The sum of the measures of the three angles in any triangle is 180°.** The two given angles have measures of 30° and 130°. The sum of the measures of these angles is 160°. Therefore, angle x must have a measure of 20° because

$$20 + 30 + 130 = 180$$

Therefore, $x =$ **20.**

example 2.2 Find x and y.

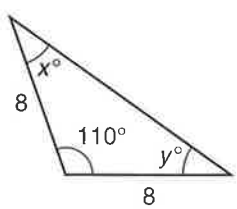


solution The identical tick marks on two sides of the triangle tell us that these two sides have equal lengths. **In any triangle, the angles opposite sides of equal lengths have equal measures.** Therefore, angle x must have a measure of 50°. So, $x =$ **50.** The sum of the measures of the three angles must be 180°. So angle y must have a measure of 80° because

$$50 + 50 + 80 = 180$$

Therefore, $y =$ **80.**

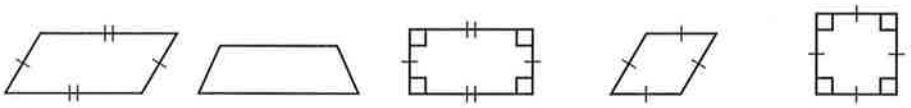
example 2.3 Find x and y.



solution The sum of the measures of the three angles must be 180°. The given angle has a measure of 110°, so the sum of the measures of angles x and y must be 70° because $180 - 110 = 70$. Angles x and y must have equal measures because the sides opposite these angles have equal lengths. Since $\frac{70}{2} = 35$, angles x and y both have measures of 35°. Therefore, $x = \mathbf{35}$ and $y = \mathbf{35}$.

2.D quadrilaterals

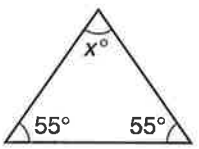
We remember that a polygon with four sides is called a quadrilateral. We will discuss five different types of quadrilaterals. A **parallelogram** is a quadrilateral that has two pairs of parallel sides. A **trapezoid** is a quadrilateral that has exactly two parallel sides. A **rectangle** is a parallelogram with four right angles. A **rhombus** is an equilateral parallelogram. A **square** is a rhombus with four right angles.



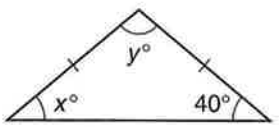
Parallelogram Trapezoid Rectangle Rhombus Square

practice

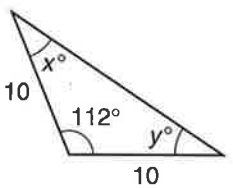
a. Find x.



b. Find x and y.



c. Find x and y.



problem set 2

†**1.** (2) What angles are formed by lines that are perpendicular?

2. (2) What angle is formed by two right angles?

3. (2) What is an acute angle?

4. (2) What is an obtuse angle?

5. (2) (a) What is the degree measure of a right angle?

(b) What is the degree measure of a straight angle?

(c) How many degrees are in a circle?

6. (2) What do you call polygons in which all sides have the same length?

†The italicized numbers within parentheses below each problem number refer to the lesson in which concepts for that problem are discussed. These numbers are explained more fully in the preface.

7. (2) What do you call polygons in which all angles have the same measure?

8. (2) What do you call polygons in which all sides have the same length and all angles have the same measure?

9. (2) (a) What is a right triangle?

(b) What is an acute triangle?

(c) What is an obtuse triangle?

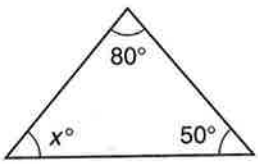
(d) What is an equiangular triangle?

10. (2) (a) What is an isosceles triangle?

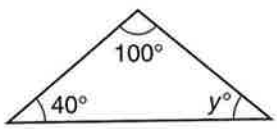
(b) What is an equilateral triangle?

(c) What is a scalene triangle?

11. (2) Find x.



12. (2) Find y.



Add or subtract as indicated. Write answers as proper fractions reduced to lowest terms or as mixed numbers.

13. (1) $\frac{1}{3} + \frac{4}{9}$

14. (1) $\frac{3}{5} + \frac{2}{7}$

15. (1) $\frac{3}{4} - \frac{5}{12}$

16. (1) $\frac{2}{3} + \frac{1}{15}$

17. (1) $\frac{9}{14} - \frac{1}{2} + \frac{3}{7}$

18. (1) $5\frac{1}{3} + 1\frac{1}{6}$

19. (1) $3\frac{1}{8} + 4\frac{1}{2}$

20. (1) $5\frac{2}{5} + 7\frac{7}{10}$

21. (1) $9\frac{1}{3} + 3\frac{3}{5}$

22. (1) $9\frac{3}{5} + 5\frac{2}{3}$

23. (1) $23\frac{7}{10} - 14\frac{2}{5}$

24. (1) $22\frac{2}{5} - 14\frac{4}{15}$

25. (1) $8\frac{2}{5} - 5\frac{1}{3}$

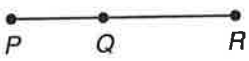
26. (1) $4\frac{2}{3} - 1\frac{5}{6}$

27. (1) $14\frac{1}{2} - 12\frac{2}{3}$

28. (1) The length of $\overline{AB}$ is $2\frac{1}{3}$ centimeters. The length of $\overline{BC}$ is $5\frac{2}{9}$ centimeters. Find AC.



29. (1) PR is $16\frac{3}{4}$ meters. QR is $9\frac{7}{8}$ meters. Find PQ.



30. (1) XZ is $10\frac{1}{5}$ units. XY is $4\frac{2}{3}$ units. Find YZ.

