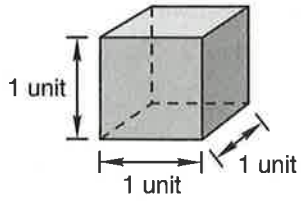
LESSON 20 Volume

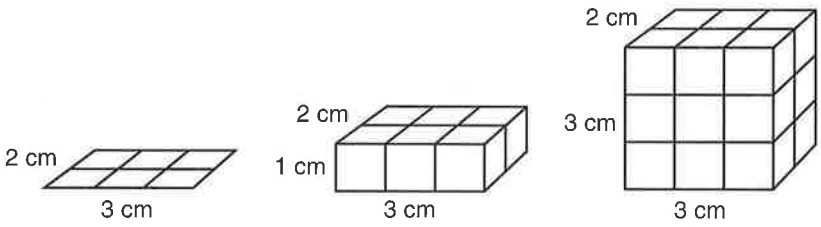
In Lesson 15 we defined a **geometric solid** as a geometric figure that has three dimensions. The **volume** of a geometric solid is the number of cubic units contained in the geometric solid. A **cubic unit** is a cube having edges that measure one unit in length. A **cube** is a geometric solid that has six identical square faces.



1 cubic unit

Volumes are always measured in cubic units. Some common units of volume are the cubic centimeter (cm^3) and the cubic meter (m^3). It is important to remember that *units* are used for length, *square units* are used for area, and *cubic units* are used for volume.

When we discuss volume, it is helpful to think of sugar cubes. We can visualize volume by mentally stacking sugar cubes. In the left-hand figure we have a rectangle that measures 3 cm by 2 cm. It has an area of 6 cm^2.

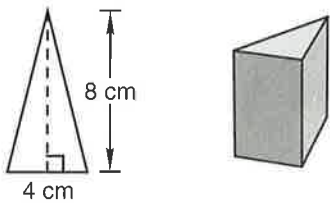


Area = 6 cm^2 Volume = 6 cm^3 Volume = 18 cm^3

In the center figure we have placed a 1-cubic-centimeter sugar cube on each square. There are 6 cubes, so we say that the volume is 6 cm^3. In the right-hand figure we have stacked the cubes three deep. There are 18 cubes, so we say that the volume is 18 cm^3. From this we can induce that the volume equals the number of cubes on the bottom layer times the number of layers. We can extend this process to determine the volume of any right solid. We remember that geometric solids whose sides are at right angles to the bases are called **right solids.** The "top" and the "bottom" of a right solid are called the **bases** of the right solid and are identical geometric figures. If the bases of a right solid are polygons, the right solid is called a **right prism.** The volume of a right solid is equal to the number of cubes that we can place on the bottom layer (area of a base) times the number of layers (height of the right solid).

> VOLUME OF A RIGHT SOLID
>
> The volume of a right solid is equal to the area of a base times the height.

example 20.1 A base of the right prism is the isosceles triangle shown. The height of the right prism is 6 cm. Find the volume of the right prism.



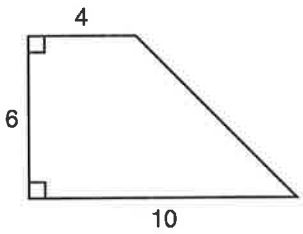
solution First we find the area of a base of the right prism. Since a base of the right prism is a triangle, the area of a base can be easily computed.

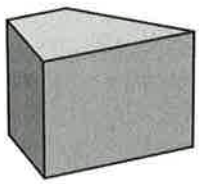
$$\text{Area of a base} = \frac{1}{2}(4\text{ cm})(8\text{ cm}) = 16\text{ cm}^2$$

The volume of a right prism is equal to the area of a base times the height. Therefore, we have

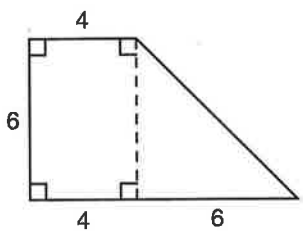
$$\begin{aligned}\text{Volume} &= (\text{area of a base})(\text{height}) \\ &= (16\text{ cm}^2)(6\text{ cm}) \\ &= \mathbf{96\text{ cm}^3}\end{aligned}$$

example 20.2 A base of the right prism 8 meters high is shown. Find the volume of the right prism. Dimensions are in meters.





solution First we find the area of a base of the right prism. The area of a base equals the sum of the areas of the rectangle and the right triangle.

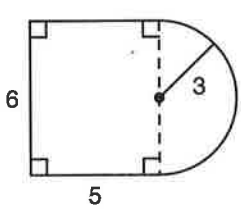


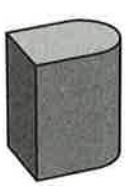
$$\text{Area of a base} = (4\text{ m})(6\text{ m}) + \frac{1}{2}(6\text{ m})(6\text{ m})$$
$$= 24\text{ m}^2 + 18\text{ m}^2$$
$$= 42\text{ m}^2$$

The volume of a right prism is equal to the area of a base times the height. Therefore, we have

$$\text{Volume} = (\text{area of a base})(\text{height})$$
$$= (42\text{ m}^2)(8\text{ m})$$
$$= \mathbf{336\text{ m}^3}$$

example 20.3 A base of the right solid 10 kilometers high is shown. Find the volume of the right solid. Dimensions are in kilometers.





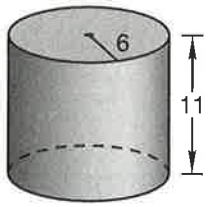
solution First we find the area of a base of the right solid. The area of a base equals the sum of the areas of the rectangle and the semicircle.

$$\text{Area of a base} = \square + \text{D}$$
$$= (5\text{ km} \times 6\text{ km}) + \frac{\pi(3\text{ km})^2}{2}$$
$$= 30\text{ km}^2 + \frac{(3.14)(3\text{ km})^2}{2}$$
$$= 30\text{ km}^2 + 14.13\text{ km}^2$$
$$= 44.13\text{ km}^2$$

The volume of a right solid is equal to the area of a base times the height. Therefore, we have

$$\begin{aligned}\text{Volume} &= (\text{area of a base})(\text{height}) \\ &= (44.13 \text{ km}^2)(10 \text{ km}) \\ &= \mathbf{441.3 \text{ km}^3}\end{aligned}$$

example 20.4 A right circular cylinder has a radius of 6 inches and a height of 11 inches, as shown. Find the volume of the right circular cylinder.



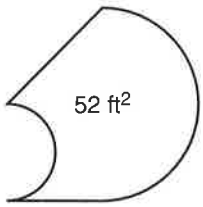
solution First we find the area of a base of the right circular cylinder. Since a base of the right circular cylinder is a circle, the area of a base can be easily computed.

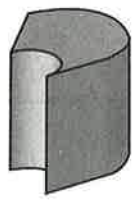
$$\text{Area of a base} = \pi(6 \text{ in.})^2 = 36\pi \text{ in.}^2$$

The volume of a right solid is equal to the area of a base times the height. Therefore, we have

$$\begin{aligned}\text{Volume} &= (\text{area of a base})(\text{height}) \\ &= (36\pi \text{ in.}^2)(11 \text{ in.}) \\ &= 396\pi \text{ in.}^3 \\ &= 396(3.14) \text{ in.}^3 \\ &= \mathbf{1243.44 \text{ in.}^3}\end{aligned}$$

example 20.5 A base of the right solid has an area of 52 ft^2. The height of the right solid is 15 ft. Find the volume of the right solid.

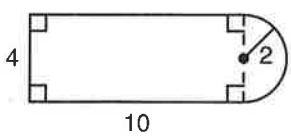




solution The volume of a right solid is equal to the area of a base times the height. Therefore, we have

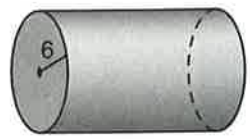
$$\begin{aligned}\text{Volume} &= (\text{area of a base})(\text{height}) \\ &= (52 \text{ ft}^2)(15 \text{ ft}) \\ &= \mathbf{780 \text{ ft}^3}\end{aligned}$$

practice **a.** A base of the right solid 10 inches high is shown. Find the volume of the right solid. Dimensions are in inches.

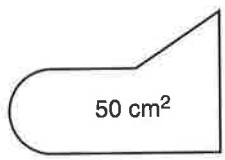


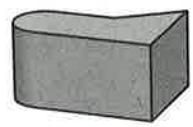


b. A right circular cylinder has a radius of 6 feet and a length of 20 feet, as shown. Find the volume of the right circular cylinder.



c. A base of the right solid has an area of 50 cm^2. The height of the right solid is 12 cm. Find the volume of the right solid.





problem set 20

1. (18) Which of the following terms are like terms?

(a) $-3xym$ (b) $2xyp$ (c) $5xyk$ (d) $4myx$

2. (6) Is the sum of a positive number and a negative number always a positive number?

3. (5) (a) Designate the set of natural numbers.

(b) Designate the set of whole numbers.

(c) Designate the set of integers.

4. (4) Use two unit multipliers to convert 50 meters to inches.

5. (10) Use two unit multipliers to convert 600 square feet to square yards.

6. (3) The perimeter of a rectangle is 110 inches. The width of the rectangle is 14 inches. Find the length of the rectangle.

7. (3) The circumference of a circle is 10π feet. Find the radius of the circle.

Simplify:

8. (19) $3^2 + (-3)^2$ **9.** (19) $-2^2 + (-4)^2$ **10.** (19) $-2^3 + (-2)^3$

11. (19) $-(-3)^2 - (-2)^3$ **12.** (19) $\sqrt[3]{-27}$ **13.** (19) $(-3)^3 - \sqrt[3]{-27}$

Evaluate:

14. (19) xz^2y^3 if $x = 2$, $z = -3$, and $y = -2$

15. (19) $a^2 - b^2a$ if $a = -2$ and $b = 3$

Simplify by adding like terms:

16. (18) $5 - x + xy - 3yx - 2 + 2x$ **17.** (18) $-3pxk + pkx - 3kpx - kp - 3kx$

Expand by using the distributive property:

18. (17) $-3(-x - 4)$ **19.** (17) $(4 - 2p)4x$ **20.** (17) $2x(a - 3p + 2)$

Evaluate:

21. (14) $-p(-a + 2p) + p$ if $p = -3$ and $a = 2$

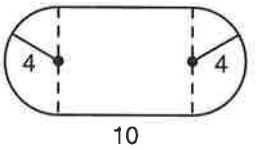
22. (14) $k(ak - 4a) + k$ if $k = -3$ and $a = 2$

23. (14) $a(x - a) + |x|$ if $a = -2$ and $x = -3$

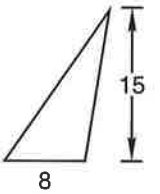
Simplify:

24. (13) $2[-3(-2 - 4)(3 - 2)]$ **25.** (12) $\dfrac{(-4)(-3 + 7)(-1)}{(7 - 4 - 1)(2 - 3)}$

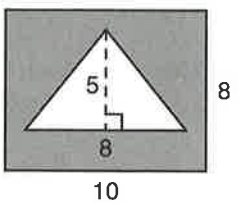
26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in centimeters.



27. (8) Find the area of this scalene triangle. Dimensions are in meters.



28. (8) Find the area of the shaded portion of this rectangle. Dimensions are in kilometers.



29. (1) WX is $5\frac{1}{2}$ inches. XY is $12\frac{3}{10}$ inches. YZ is $3\frac{2}{5}$ inches. Find WZ.



30. (20) A base of the right solid has an area of 45 ft^2. The height of the right solid is 10 ft. Find the volume of the right solid.

