LESSON 21 *Product Rule for Exponents • Addition of Like Terms with Exponents*

21.A product rule for exponents

The product rule for exponents can be deduced from the definition of exponential notation. We know that

(a) 3^5 means $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

and thus (b) $3^2 \cdot 3^3$ means $(3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)$, or 3^5

and also (c) $3 \cdot 3^4$ means $(3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$, or 3^5

We see that when we multiply exponentials whose bases are the same, the exponent of the product is obtained by adding the exponents of the factors. Thus,

(a) $x^5 \cdot x^7 \cdot x^2 = x^{14}$ (b) $5^2 \cdot 5^3 \cdot 5^2 = 5^7$

(c) $p^5 \cdot p^{12} = p^{17}$ (d) $4^2 \cdot 4^3 \cdot 4^{25} = 4^{30}$

We call this rule the **product rule for exponents** and give the formal definition as follows:

PRODUCT RULE FOR EXPONENTS

If m and n are real numbers and $x \neq 0$, then

$$x^m \cdot x^n = x^{m+n}$$

We can use this rule to help simplify expressions that contain exponents.

example 21.1 Simplify: $x^2y^2x^5y^3$

solution Since rearranging the order of the factors of a product does not change the value of the product, we may write $x^2y^2x^5y^3$ as

$$x^2x^5y^2y^3 = \boldsymbol{x^7y^5}$$

example 21.2 Simplify: $x^2y^3m^5x^3y^2$

solution First we rearrange the factors and then we simplify.

$$x^2x^3y^2y^3m^5 = \boldsymbol{x^5y^5m^5}$$

Now we define the notation x^1.

DEFINITION

$$x^1 = x$$

This says that x means the same thing as x^1 and that x^1 means the same thing as x. **If any variable or constant is written without an exponent, it is understood to have an exponent of 1.** Thus,

5^1 equals 5, 7^1 equals 7, and x^1 equals x

example 21.3 Simplify: xyy^2x^3

solution First we rearrange the letters. Then we simplify, remembering that x means x^1 and y means y^1.

$$xx^3yy^2 = \boldsymbol{x^4y^3}$$

example 21.4 Simplify: $m^3pmxm^2x^3p^5$

solution We rearrange and simplify to get the result.

$$m^3mm^2xx^3pp^5 = \boldsymbol{m^6x^4p^6}$$

We note that there are six ways that this result can be written:

$$m^6x^4p^6 \quad m^6p^6x^4 \quad x^4m^6p^6 \quad x^4p^6m^6 \quad p^6m^6x^4 \quad p^6x^4m^6$$

Because the order of factors does not affect the product, all six of these answers are equally correct and none is preferred. In mathematics, preference is reserved to the individual. There is no requirement to alphabetize the variables.

21.B addition of like terms with exponents

In the previous section we noted that we multiply exponential expressions with like bases by adding the exponents. The task of adding like terms that contain exponents appears similar, but the rule is different. When we add like terms that contain exponents, we do not add the exponents. Thus

$$3x^2 + 2x^2 = 5x^2$$

and does not equal $5x^4$. Addition and multiplication are often confused, so we discuss them in the same lesson so that we can point out the difference.

When we add, we can only add like terms. We recall that letters stand for unspecified numbers and that the order of multiplication of real numbers can be changed. This means that

$$x^2yp^5 \quad \text{and} \quad p^5x^2y$$

are like terms, and that

$$xy^2p^5 \quad \text{and} \quad y^2xp^5$$

are like terms because the literal factors of the terms are the same.

example 21.5 Simplify by adding like terms: $x^2yp^5 + 2xy^2p^5 + 3p^5x^2y - 7y^2xp^5$

solution The first and third terms are like terms, and the second and fourth terms are also like terms. We add the numerical coefficients of these terms and get

$$\mathbf{4x^2yp^5 - 5xy^2p^5}$$

example 21.6 Simplify by adding like terms: $2m^3xy^2p + 3pxy^2m^3 - 10xy^2m^3p + yx^2m^3p$

solution The first three terms are like terms and may be added.

$$\mathbf{-5xy^2m^3p + yx^2m^3p}$$

example 21.7 Simplify by adding like terms: $2x^2y + 3yx^2 + x^2y^2 - x^2y - 4x^2y^2$

solution The first, second, and fourth terms are like terms and so are the third and fifth terms. Thus we add the first, second, and fourth terms and the third and fifth terms to obtain

$$\mathbf{4x^2y - 3x^2y^2}$$

practice Simplify:

a. $xyx^4x^3y^5$

b. $x^3xy^2y^5x^7mm$

Simplify by adding like terms:

c. $2x^2y^3 + xy - 8y^3x^2 - 5yx$

d. $x^6y + yx^6 + 3xy - 5xy^6$

problem set 21

1. (13) Is the product of 13 negative numbers and 10 positive numbers a positive number or a negative number?

2. (17) Use the letters a, b, and c and parentheses to write the distributive property.

3. (2) (a) What is an isosceles triangle?

(b) What is an equilateral triangle?

(c) What is a scalene triangle?

4. (4) Use two unit multipliers to convert 366 centimeters to feet.

5. (10) Use two unit multipliers to convert 5000 square meters to square kilometers.

6. (8) The area of a rectangle is 95 square centimeters. The width of the rectangle is 5 centimeters. Find the length of the rectangle.

7. (8) The radius of a circle is 10 meters. Find the area of the circle.

Simplify:

8. (21) x^2yyyx^3yx

9. (21) $xm^2xm^3x^3m$

10. (21) $ky^2k^3k^2y^5$

11. (21) $a^2ba^2b^3ab^4$

Simplify by adding like terms:

12. (18) $4xyz - 3yz + zxy$

13. (18) $7 - 3k - 2k + 2kx - xk + 8$

14. (21) $3ab^2 - 2ab + 5b^2a - ba$

15. (21) $x^2 - 3yx + 2yx^2 - 2xy + yx$

Expand by using the distributive property:

16. (17) $5(2 - 4p)$

17. (17) $x(3p - 2y)$

18. (17) $(3 - 2b)a$

Evaluate:

19. (14) $(a - x)(x - a)$ if $a = -3$ and $x = 4$

20. (14) $m(x - m) - |x|$ if $m = -3$ and $x = -4$

21. (19) $x^2 - y^2$ if $x = -3$ and $y = -2$

Simplify:

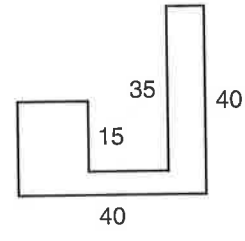
22. (19) $-5 - (-5)^2 - 3 + (-2)$

23. (19) $-3^2 - 2^2 - (-3)^3 - \sqrt[3]{-8}$

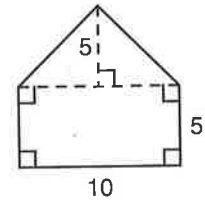
24. (13) $\dfrac{3 - [-(-3)]}{-(-2)}$

25. (12) $\dfrac{5(-6 + 4) + 7(-3 + 9)}{(5 - 3) + 6}$

26. (3) Find the perimeter of this figure. All angles are right angles. Dimensions are in inches.



27. (8) Find the area of this figure. Dimensions are in feet.



Simplify:

28. (9) $\dfrac{-1\frac{3}{4}}{2\frac{1}{3}}$

29. (4) $(0.004)(0.012)$

30. (15) A right circular cylinder has a radius of 3 yards and a length of 10 yards, as shown. Find the surface area of the right circular cylinder.

