LESSON 22 *Review of Numerical and Algebraic Expressions • Statements and Sentences • Conditional Equations*

22.A review of numerical and algebraic expressions

Each of the following expressions

$$\text{(a) } 4 \qquad \text{(b) } 6 + 3 \qquad \text{(c) } 4(2 + 3) \qquad \text{(d) } \frac{7(8 + 4)}{5 + 2}$$

is called a **numerical expression** because it consists of **a meaningful arrangement of numerals and symbols that designate specific operations.** Every numerical expression represents a particular number, and we say that this number is the **value** of the expression. The **values** of expressions (a), (b), (c), and (d) are 4, 9, 20, and 12, respectively.

We use the words **algebraic expression** to describe **numerical expressions** and also to describe expressions that contain variables.

$$\text{(e) } 6 \qquad \text{(f) } x + 4 \qquad \text{(g) } x^2 - 6 \qquad \text{(h) } x(x + 4)$$

Each of the expressions shown in (e) through (h) is an algebraic expression. The value of expression (e) is 6, but the values of expressions (f), (g), and (h) depend on the value that we assign to the variable x. If we assign to x the value of 3, then the values of expressions (f), (g), and (h) are 7, 3, and 21, respectively.

22.B statements and sentences

If we wish to make a statement that certain quantities are equal or not equal, we can do so by writing a grammatical **sentence** in English.

(a) The number of peaches equals the number of apples.

(b) The number of peaches does not equal the number of apples.

(c) The number of peaches is greater than the number of apples.

(d) The number of peaches is less than the number of apples.

If we use the variables

$$N_p \qquad \text{and} \qquad N_a$$

to represent the number of peaches and the number of apples, and if we use the symbols

$=$	to mean	equal
$\neq$	to mean	not equal
$>$	to mean	greater than (read left to right)
$<$	to mean	less than (read left to right)

we can make the same statements by writing algebraic sentences.

$$\text{(a) } N_p = N_a$$

$$\text{(b) } N_p \neq N_a$$

$$\text{(c) } N_p > N_a$$

$$\text{(d) } N_p < N_a$$

We see that all four are called statements or sentences but only the first one, (a), uses the equals sign. This algebraic statement is called an **equation.** The other three statements, (b), (c), and (d), do not use the equals sign and are called **inequalities.** We will discuss equations here and hold the topic of inequalities for a later lesson.

22.C conditional equations

An *equation* is an algebraic statement consisting of two algebraic expressions connected by an equals sign. Thus all the following are statements and all are also equations.

(a) $4 = 3 + 1$ (b) $4 + x = 2 + 2 + x$ (c) $4 = 6$

(d) $4 + x = 6 + x$ (e) $x + 4 = 8$

Equations are not always true equations, as we see here. Two of these equations are true equations and two are false equations, and the truth or falsity of one of them depends on the number we use as a replacement for x in the equation.

(a) This is a true equation because 4 does equal the sum of 3 and 1.

(b) This is a true equation regardless of the number we use as a replacement for x. We will demonstrate this by replacing x with –3 and then by replacing x with +7.

WITH (–3)		WITH (+7)	
(b) $4 + (-3) = 2 + 2 + (-3)$		(b) $4 + (+7) = 2 + 2 + (+7)$	
$4 - 3 = 4 - 3$		$4 + 7 = 4 + 7$	
$1 = 1$	True	$11 = 11$	True

(c) This is a false equation because 4 is not equal to 6.

(d) This is a false equation regardless of the number we use as a replacement for x. We will demonstrate this by replacing x with –3 and then by replacing x with +7.

WITH (–3)		WITH (+7)	
(d) $4 + (-3) = 6 + (-3)$		(d) $4 + (+7) = 6 + (+7)$	
$4 - 3 = 6 - 3$		$4 + 7 = 6 + 7$	
$1 = 3$	False	$11 = 13$	False

(e) We call this equation a **conditional equation** because its truth or falsity is conditioned by the number used as a replacement for x. If we use –2 as the replacement for x, we get a false equation; but if we use 4 as the replacement for x, we get a true equation.

WITH (–2)		WITH (4)	
(e) $(-2) + 4 = 8$		(e) $(4) + 4 = 8$	
$-2 + 4 = 8$		$4 + 4 = 8$	
$2 = 8$	False	$8 = 8$	True

Replacement values of the variable that turn the equation into a true equation are called *solutions* of the equation or *roots* of the equation and are said to satisfy the equation. Thus in the equation

$$x + 4 = 8$$

we say that the number 4 is a **solution** or **root** of the equation, and we also say that the number 4 **satisfies** the equation.

example 22.1 Does –2 or +5 satisfy the equation $x^2 = -5x - 6$?

solution First we try –2.

$(-2)^2 = -5(-2) - 6$	replaced x with –2
$4 = 10 - 6$	simplified
$4 = 4$	True

Now we try +5.

$$(5)^2 = -5(5) - 6 \qquad \text{replaced } x \text{ with } 5$$
$$25 = -25 - 6 \qquad \text{simplified}$$
$$25 = -31 \qquad \text{False}$$

Thus **–2 is a solution** and **+5 is not a solution.**

example 22.2 Is –2 or +5 a root of the equation $x^2 - 3x = 10$?

solution First we try –2.

$$(-2)^2 - 3(-2) = 10 \qquad \text{replaced } x \text{ with } -2$$
$$4 + 6 = 10 \qquad \text{simplified}$$
$$10 = 10 \qquad \text{True}$$

Now we try +5.

$$(5)^2 - 3(5) = 10 \qquad \text{replaced } x \text{ with } 5$$
$$25 - 15 = 10 \qquad \text{simplified}$$
$$10 = 10 \qquad \text{True}$$

Thus, both –2 and +5 are solutions or roots to the given equation, and we say that **both –2 and +5 satisfy this equation.**

practice

a. Does –2 or 2 satisfy the equation $x - 2 = 0$?

b. Is –2 or –5 a root of the equation $x^2 + 7x = -10$?

problem set 22

1. *(22)* (a) What is an equation?

(b) What is a conditional equation?

2. *(22)* What do we call the replacement values of the variable that turn an equation into a true equation?

3. *(18)* Which of the following terms are like terms?

(a) $-aby$ (b) $8yba$ (c) $3abm$ (d) $-6bya$

4. *(4)* Use one unit multiplier to convert 72 inches to centimeters.

5. *(10)* Use two unit multipliers to convert 55 square meters to square centimeters.

6. *(3)* The perimeter of a rectangle is 124 centimeters. Two of the sides are each 45 centimeters long. What is the length of each of the other two sides?

7. *(8)* The area of a circle is 4π square meters. Find the radius of the circle.

8. *(22)* Does –1 or 1 satisfy the equation $x - 1 = 0$?

9. *(22)* Is –3 or 2 a root of the equation $x^2 - x = 12$?

Simplify:

10. *(21)* $x^2xxy^2xy^3$

11. *(21)* $a^2aba^3b^2a^5$

12. *(21)* $p^2m^5ypp^3my^2$

13. *(21)* $4p^2x^2kpx^3k^2k$

Simplify by adding like terms:

14. (18) $-8 - py + 2yp + 4 - y$

15. (18) $m + 4 + 3m - 6 - 2m + mc - 4mc$

16. (21) $xy - 3xy^2 + 5y^2x - 4xy$

17. (21) $-3x^2ym + 7x - 5ymx^2 + 16x$

Expand by using the distributive property:

18. (17) $a(3x - 2)$

19. (17) $4xy(5 - 2a)$

20. (17) $2x(4a + b - 3m)$

Evaluate:

21. (14) $cy(cx - y)$ if $x = -3$, $y = 3$, and $c = -2$

22. (14) $|x - a| - a(-x)$ if $x = 4$ and $a = 3$

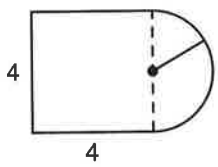
23. (19) $a - b(a^2 - b)$ if $a = -2$ and $b = 3$

Simplify:

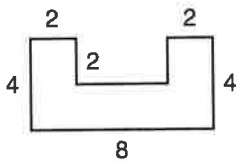
24. (19) $(-3)^3 + (-2)^3 - |-2|$

25. (19) $-3^2 - (-2)^2 + \sqrt[3]{-27}$

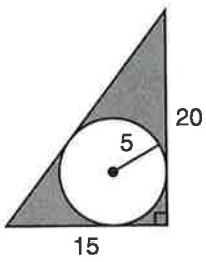
26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in kilometers.



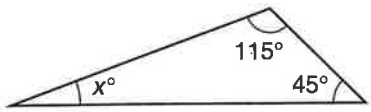
27. (8) Find the area of this figure. All angles are right angles. Dimensions are in inches.



28. (8) Find the area of the shaded portion of this right triangle. Dimensions are in feet.



29. (2) Find x.



30. (20) A base of the right prism is the isosceles triangle shown. The height of the right prism is 10 in. Find the volume of the right prism.

