LESSON 23 Equivalent Equations • Additive Property of Equality

23.A equivalent equations

Two equations are said to be equivalent if *every* solution of either one of the equations is also a solution of the other equation.

$$\text{(a) } x + 6 = 9 \qquad \text{(b) } x + 10 = 13$$

The two equations shown are equivalent equations, for the number 3 will satisfy both equations and 3 is the only number that will satisfy either equation.

23.B additive property of equality

If we begin with the true statement that

$$2 = 2$$

and add +4 to both sides of the equality, we get the true statement that 6 equals 6.

$$2 + 4 = 2 + 4 \qquad 4 + 2 = 4 + 2$$

$$6 = 6 \qquad 6 = 6$$

On the left we placed the 4s after the 2s and on the right we placed the 4s before the 2s. We note that both procedures yield the same result. We emphasize this fact in the formal definition given in the box by writing the definition twice, the second time with the order of the addends reversed.

> ADDITIVE PROPERTY OF EQUALITY
>
> If a, b, and c are any real numbers and if $a = b$, then
>
> $a + c = b + c$ and also $c + a = c + b$

The additive property of equality can be used to find the solution of conditional equations such as

$$x + 4 = 6$$

This equation is a conditional equation and is neither true nor false because no value has been assigned to x, so the additive property of equality does not apply. Thus we must hedge a little. We assume that some real number exists that when substituted for x will make the equation a true equation. We further assume that x in the equation represents this number. Now we have assumed that

$$x + 4 \text{ equals } 6$$

is a true statement, and thus we can use the additive property of equality. We will use the additive property of equality to eliminate the +4 that is now on the left side with x. We will add −4 to both sides of the equation.

$$\begin{array}{rl} x + 4 = 6 & \text{original equation} \\ \quad -4 \quad -4 & \text{added } -4 \text{ to both sides} \\ \hline x + 0 = 2 & \end{array}$$

$$x = 2$$

Now since we made an assumption to permit the use of the additive property of equality, we must check our solution in the original equation.

$$\begin{aligned} x + 4 &= 6 && \text{original equation} \\ (2) + 4 &= 6 && \text{substituted 2 for } x \\ 6 &= 6 && \text{True} \end{aligned}$$

Since using the number 2 for x in the equation makes the equation a true equation, we say that the number 2 is a solution or root of the equation and that the number 2 satisfies the equation. It can be shown that the use of the additive property of equality will not change the numbers that satisfy the equation, so we say that the use of the additive property of equality results in an equation that is an **equivalent equation** to the original equation.

> If the same quantity is added to both sides of an equation, the resulting equation will be an **equivalent equation** to the original equation, and thus every solution of one of these equations will also be a solution of the other equation.

The rule says that we may add the same quantity to both sides of an equation, but it does not specify a particular format to be used, and one format is usually just as acceptable as another. Below are shown three possible formats for the problem worked previously.

(a) $\begin{aligned} x + 4 &= 6 \\ -4 &= -4 \\ \hline x &= 2 \end{aligned}$ (b) $\begin{aligned} x + 4 &= 6 \\ -4 &\quad -4 \\ \hline x &= 2 \end{aligned}$ (c) $\begin{aligned} x + 4 + (-4) &= 6 + (-4) \\ x + 0 &= 2 \\ x &= 2 \end{aligned}$

In (a) we added -4 to both sides of the equation and placed an equals sign between the -4s to *emphasize* that they are equal. In (b) we added -4 to both sides of the equation but omitted the equals sign since it is unnecessary. In (c) we added the -4s on the same line with the rest of the numbers and variables. This form is adequate for very simple problems such as this one but is less desirable for more complicated problems. By the end of the book, one should be able to perform this calculation mentally, without writing anything down, so that none of these formats will be necessary.

example 23.1 Solve: $x - 3 = 12$

solution To solve the equation, we want to isolate x on one side of the equals sign. We can do this if we eliminate the -3. Thus we will add $+3$ to both sides of the equation.

$$\begin{aligned} x - 3 &= 12 \\ +3 &\quad +3 \\ \hline x &= \mathbf{15} \end{aligned}$$

This same procedure can be used when the equation contains fractions or mixed numbers, as we see in the next two examples.

example 23.2 Solve: $x + \frac{1}{4} = -\frac{3}{8}$

solution To isolate x we must eliminate the $\frac{1}{4}$. Thus, we add $-\frac{1}{4}$ to both sides of the equation.

$$\begin{aligned} x + \frac{1}{4} &= -\frac{3}{8} \\ -\frac{1}{4} &\quad -\frac{1}{4} \\ \hline x &= -\frac{3}{8} - \frac{1}{4} \end{aligned} \quad \rightarrow \quad x = -\frac{\mathbf{5}}{\mathbf{8}}$$

example 23.3 Solve: $k + 2\frac{1}{3} = \frac{2}{9}$

solution This time we add $-2\frac{1}{3}$ to both sides of the equation and then simplify the result.

$$\begin{array}{rcl} k + 2\frac{1}{3} & = & \frac{2}{9} \\ -2\frac{1}{3} & & -2\frac{1}{3} \\ \hline k & = & \frac{2}{9} - 2\frac{1}{3} \end{array} \quad \rightarrow \quad k = \frac{2}{9} - \frac{21}{9} \quad \rightarrow \quad k = \mathbf{-\frac{19}{9}}$$

practice Solve:

a. $x + 5 = 17$

b. $k - 27 = -38$

c. $x - \frac{1}{2} = \frac{3}{8}$

d. $d + 4\frac{1}{7} = 3\frac{1}{6}$

problem set 23

1. (23) What does "solve an equation" mean?

2. (23) What are equivalent equations?

3. (2) What is the sum of the measures of the three angles of any triangle?

4. (4) Use one unit multiplier to convert 150 meters to centimeters.

5. (10) Use two unit multipliers to convert 116 square inches to square centimeters.

6. (8) The area of a rectangle is 209 square inches. The length of one side is 19 inches. What is the length of the other side?

7. (8) The area of a circle is 9π square feet. Find the radius of the circle.

Solve:

8. (23) $x - 4 = 10$

9. (23) $x + \frac{1}{5} = -\frac{1}{10}$

10. (23) $x + 1\frac{1}{4} = -\frac{5}{8}$

11. (22) Does -2 or 2 satisfy the equation $x + 2 = 0$?

12. (22) Is -2 or 3 a root of the equation $x^2 - 2x = 3$?

Simplify:

13. (21) $x^2ym^5x^2y^4$

14. (21) x^3y^2myxm

15. (21) xxx^2yyy^3xy

Simplify by adding like terms:

16. (18) $-4 + 7x - 3x - 5 + 2x - 4x$

17. (21) $p^2xy - 3yp^2x + 2xp^2y - 5$

Expand by using the distributive property:

18. (17) $4x(a + 2b)$

19. (17) $(2x + 4)3$

20. (17) $4px(my - 3ab)$

Evaluate:

21. (14) $x(x - y) - y$ if $x = -2$ and $y = 3$

22. (14) $(a - b)(b - x)$ if $x = 2$, $a = 3$, and $b = -3$

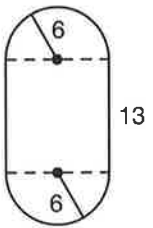
23. (19) $x^3 - a(a - b)$ if $x = -2$, $a = 3$, and $b = 3$

Simplify:

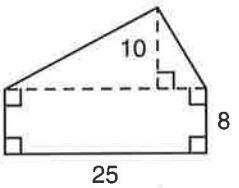
24. (19) $-3^2 - (-3)^2 + \sqrt[5]{32}$

25. (12) $\dfrac{-3 - 4(4) - 6}{10 + (-5)(-4)}$

26. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in yards.



27. (8) Find the area of this figure. Dimensions are in centimeters.



Simplify:

28. (4) $3\frac{3}{7} \times 3\frac{1}{16}$

29. (9) $\dfrac{3\frac{1}{5}}{-2\frac{2}{15}}$

30. (15) Find the surface area of this right rectangular prism. Dimensions are in meters.

