LESSON 24 Multiplicative Property of Equality

To demonstrate the **multiplicative property of equality,** we will begin with the true equation

$$2 = 2 \qquad \text{true}$$

and multiply both sides of the equation by 3.

$$3 \cdot 2 = 3 \cdot 2$$

The result is the true equation that 6 equals 6.

$$6 = 6 \qquad \text{still true}$$

The formal statement of the multiplicative property of equality in the box below is made twice to emphasize the fact that the order of the factors does not affect the product.

MULTIPLICATIVE PROPERTY OF EQUALITY

If a, b, and c are any real numbers and if

$$a = b,$$

then

$$ca = cb \qquad \text{and also} \qquad ac = bc$$

It is possible to use the multiplicative property of equality to prove that multiplying or dividing every term on both sides of an equation by the same nonzero number does not change the solution(s) to the equation. This means that the new equation is an equivalent equation to the original equation.

> If **every term** on both sides of an equation is multiplied or divided by the same nonzero quantity, the resulting equation will be an equivalent equation to the original equation, and thus every solution of one of these equations will be a solution of the other equation.

We can use this rule with either of two thought processes to solve equations such as

$$4x = 12$$

The first way is to remember that division by 4 will undo multiplication by 4 because division and multiplication are inverse operations. Thus, we solve by dividing both sides of the equation by 4.

$$4x = 12 \rightarrow \frac{4x}{4} = \frac{12}{4} \rightarrow x = 3$$

The second way is to remember that the product of 4 and its reciprocal is 1. To solve by using this thought, we will multiply both sides of the equation by $\frac{1}{4}$, which is the reciprocal of 4.

$$4x = 12 \rightarrow \frac{1}{4} \cdot 4x = \frac{1}{4} \cdot 12 \rightarrow x = 3$$

Both of the preceding thought processes are correct and either one can be used at any time.

Example 24.1 Solve: $5x = 20$

solution We can solve by (a) multiplying both sides of the equation by $\frac{1}{5}$, or by (b) dividing both sides of the equation by 5.

(a) $\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 20$

$x = \mathbf{4}$

(b) $\frac{5x}{5} = \frac{20}{5}$

$x = \mathbf{4}$

Example 24.2 Solve: $\frac{2}{5}x = 7$

solution We can solve by (a) multiplying both sides of the equation by $\frac{5}{2}$, or by (b) dividing both sides of the equation by $\frac{2}{5}$.

(a) $\frac{5}{2} \cdot \frac{2}{5}x = \frac{5}{2} \cdot 7$

$x = \mathbf{\frac{35}{2}}$

(b) $\frac{\frac{2}{5}x}{\frac{2}{5}} = \frac{7}{\frac{2}{5}}$

$x = 7 \cdot \frac{5}{2} \rightarrow x = \mathbf{\frac{35}{2}}$

Many beginning algebra students would prefer to write the preceding answer as the mixed number $17\frac{1}{2}$. Both forms of the answer are equally correct, but we prefer the improper fraction $\frac{35}{2}$ to the mixed number $17\frac{1}{2}$ because the improper fraction is easier to use. Suppose the instructions in this problem had been to solve and then multiply the answer by $\frac{11}{4}$. If we

had written the answer as the mixed number $17\frac{1}{2}$, we would have to change it back to the improper fraction $\frac{35}{2}$ before the multiplication could be performed. Instructors at this level and in higher courses usually prefer the improper fraction to the mixed number.

example 24.3 Solve: $2\frac{1}{4}x = 3$

solution We can undo multiplication by $2\frac{1}{4}$ by dividing both sides of the equation by $2\frac{1}{4}$,

$$2\tfrac{1}{4}x = 3 \rightarrow \frac{2\frac{1}{4}x}{2\frac{1}{4}} = \frac{3}{2\frac{1}{4}} \rightarrow x = \frac{3}{\frac{9}{4}} \rightarrow x = \frac{12}{9} = \mathbf{\frac{4}{3}}$$

or by rewriting $2\frac{1}{4}$ as the improper fraction $\frac{9}{4}$ and then multiplying both sides of the equation by $\frac{4}{9}$, which is the reciprocal of $\frac{9}{4}$.

$$\frac{9}{4}x = 3 \rightarrow \frac{4}{9}\cdot\frac{9}{4}x = \frac{4}{9}\cdot 3 \rightarrow x = \mathbf{\frac{4}{3}}$$

example 24.4 Solve: $\frac{x}{3} = 9$

solution We can undo division by 3 by multiplying both sides of the equation by 3. Thus, we multiply both sides of the equation by 3 and cancel.

$$3\cdot\frac{x}{3} = 9\cdot 3 \qquad \text{and thus} \qquad x = \mathbf{27}$$

We can also use the concept of inverse operations when x is divided by a fraction or a mixed number.

example 24.5 Solve: (a) $\frac{x}{2\frac{1}{2}} = 7$ (b) $\frac{p}{\frac{3}{2}} = 4\frac{1}{3}$

solution (a) We can undo division by $2\frac{1}{2}$ by multiplying both sides of the equation by $2\frac{1}{2}$.

$$\frac{x}{2\frac{1}{2}} = 7 \rightarrow \frac{2\frac{1}{2}x}{2\frac{1}{2}} = 7\cdot 2\tfrac{1}{2} \rightarrow x = 7\cdot\frac{5}{2} \rightarrow x = \mathbf{\frac{35}{2}}$$

(b) We can undo division by $\frac{3}{2}$ by multiplying both sides of the equation by $\frac{3}{2}$.

$$\frac{p}{\frac{3}{2}} = 4\tfrac{1}{3} \rightarrow \frac{\frac{3}{2}p}{\frac{3}{2}} = 4\tfrac{1}{3}\cdot\frac{3}{2} \rightarrow p = \frac{13}{3}\cdot\frac{3}{2} = \mathbf{\frac{13}{2}}$$

practice Solve:

a. $\frac{3}{5}x = 27$ **b.** $3\frac{1}{5}y = 32$ **c.** $\frac{x}{\frac{1}{4}} = 20$ **d.** $\frac{x}{2\frac{1}{4}} = 5$

problem set 24

1. *(13)* Is the product of 16 negative numbers and 11 positive numbers a positive number or a negative number?

2. *(18)* Which of the following terms are like terms?

(a) $-4xyk$ (b) $-4xyz$ (c) $7zyx$ (d) $7zyk$

3. (2) (a) What are the measures of the angles of an equiangular triangle?

(b) What are the measures of the angles of an equilateral triangle?

4. (4) Use two unit multipliers to convert 280 miles to inches.

5. (10) Use two unit multipliers to convert 45 square feet to square inches.

6. (8) The length of the base of a right triangle is 24 centimeters. The height of the right triangle is 18 centimeters. Find the area of the right triangle.

7. (3) The circumference of a circle is 12π meters. Find the radius of the circle.

Solve:

8. (23) $x + 5 = 7$

9. (23) $y - 3 = 2$

10. (23) $x - \frac{1}{4} = \frac{7}{8}$

11. (23) $y - \frac{1}{2} = -2\frac{1}{2}$

12. (23) $x + \frac{1}{2} = 2\frac{1}{5}$

13. (24) $2x = 20$

14. (24) $3x = 4\frac{1}{2}$

15. (24) $\frac{x}{3} = 5$

16. (24) $\frac{x}{\frac{1}{2}} = 4$

17. (22) Does -5 or 5 satisfy the equation $x - 5 = 0$?

Simplify:

18. (21) $m^2xyp^2x^3y^5$

19. (21) $3p^2xxypp^3xy^2$

Simplify by adding like terms:

20. (18) $a + 3 - 2a - 5a + 5 - a$

21. (21) $-3x^2ym + 5myx^2 - 2my^2x$

22. (17) Use the distributive property to expand $(3a - 5p)xy$.

Evaluate:

23. (19) $(-x)^3 - y$ if $x = -2$ and $y = 4$

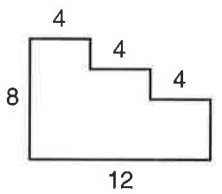
24. (19) $a^3 - (a - b) + |a - b|$ if $a = -2$ and $b = 3$

Simplify:

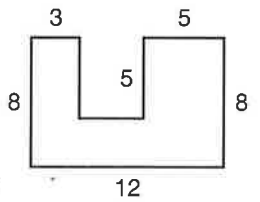
25. (13) $-2\{-5(-3 + 4) - [3 - (-4)]\}$

26. (19) $(-2)^3 - (-2)^2 - 5 + \sqrt[3]{-64}$

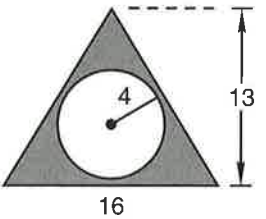
27. (3) Find the perimeter of this figure. All angles are right angles. Dimensions are in kilometers.



28. (8) Find the area of this figure. All angles are right angles. Dimensions are in inches.



29. (8) Find the area of the shaded portion of this isosceles triangle. Dimensions are in feet.



30. *(20)* A base of the right solid has an area of 95 cm^2. The height of the right solid is 16 cm. Find the volume of the right solid.

