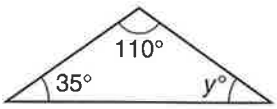
**16.** (2) Find $y$.




Add or subtract as indicated. Write answers as proper fractions reduced to lowest terms or as mixed numbers.

**17.** (1) $\frac{1}{2} + \frac{5}{18}$

**18.** (1) $\frac{11}{12} - \frac{3}{4}$

**19.** (1) $\frac{8}{15} + \frac{2}{3} - \frac{1}{5}$

**20.** (1) $5\frac{2}{3} + 1\frac{7}{12}$

**21.** (1) $7\frac{5}{6} + 4\frac{1}{18}$

**22.** (1) $6\frac{3}{5} + 14\frac{9}{10}$

**23.** (1) $4\frac{7}{8} + 3\frac{3}{16}$

**24.** (1) $5\frac{1}{8} + 8\frac{3}{7}$

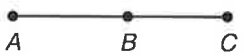
**25.** (1) $4\frac{3}{5} - 3\frac{4}{15}$

**26.** (1) $15\frac{2}{3} - 4\frac{5}{11}$

**27.** (1) $7\frac{5}{6} - 6\frac{11}{12}$

**28.** (1) $33\frac{5}{8} - 7\frac{11}{16}$

**29.** (1) The length of $\overline{AB}$ is $14\frac{1}{3}$ feet. The length of $\overline{BC}$ is $12\frac{2}{5}$ feet. Find $AC$.



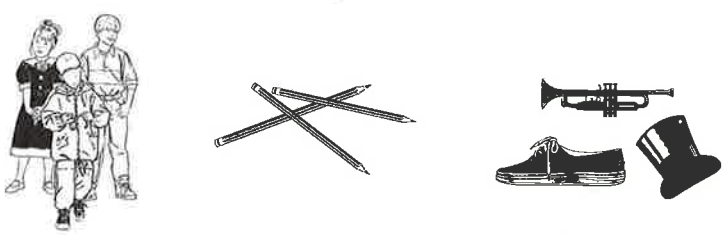

**30.** (1) $DF$ is $7\frac{2}{3}$ yards. $EF$ is $4\frac{1}{2}$ yards. Find $DE$.




# *LESSON 4* *Review of Arithmetic*

## 4.A numerals and numbers

**A *number* is an idea. A *numeral* is a single symbol or a collection of symbols that we use to express the idea of a particular number.**



The three drawings above all have the quality of *threeness*. The three children and the three pencils both bring to mind the idea of *three*. The drawing at the right also brings to mind the idea of *three*, although all the things in the drawing are not of the same kind.

If we wish to use a symbol to designate the idea of three, we could write any of the following:

$$\text{III}, \quad 3, \quad \frac{30}{10}, \quad \frac{27}{9}, \quad \frac{33}{11}, \quad 2 + 1, \quad 6 \div 2, \quad 11 - 8$$

Each of these is a symbolic representation of the idea of three. Throughout the book, when we use the word *number*, we are describing the idea. We will use numerals to designate the

numbers. But we will remember that none of the marks we make on paper are numbers because

**A number is an idea!**

Since the symbols

$$3 \quad \text{and} \quad \frac{30}{10}$$

are both numerals that represent the same number, we say that they have the same value. **Thus, the value of a numeral is the number represented by the numeral, and we see that the words *value* and *number* have the same meaning.**

## 4.B natural or counting numbers

The system of numeration that we use to designate numbers is called the **decimal system.** It was invented by the Hindus of India, passed to their Arab neighbors, and finally transmitted to Europe circa A.D. 1200. The decimal system uses 10 symbols that we call *digits*. These digits are

$$0, \quad 1, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6, \quad 7, \quad 8, \quad 9$$

We use these digits by themselves or in combination with one another to form the numerals that we use to designate decimal numbers.

**We call the numbers that we use to count objects or things the *natural numbers* or the *counting numbers*.** When we begin counting, we always begin with the number 1 and follow it with the number 2, etc.

$$1, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6, \quad 7, \quad 8, \quad 9, \quad 10, \quad 11, \quad 12, \quad 13, \ldots$$

It would not be natural to try to count by using numbers such as $\frac{1}{2}$ or 0 or $\frac{3}{4}$, so these numbers are not called natural or counting numbers. We designate the natural or counting numbers with the list above. The three dots after the number 13 indicate that this list continues without end.

## 4.C real numbers

The numbers of arithmetic are zero and the positive real numbers. **We say that a *positive real number* is any number that can be used to describe a physical distance greater than zero.** Thus, all the numbers shown here

$$\frac{3}{4} \qquad 0.000163 \qquad 363 \qquad 3\frac{3}{8} \qquad 46 \qquad \frac{11}{7} \qquad 400.1623232323$$

are positive real numbers, for all of them can be used to describe physical distances when used with descriptive units such as inches, feet, yards, etc.

$$\frac{3}{4} \text{ mile} \qquad 0.000163 \text{ yard} \qquad 363 \text{ feet} \qquad 3\frac{3}{8} \text{ meters}$$

$$46 \text{ inches} \qquad \frac{11}{7} \text{ kilometers} \qquad 400.1623232323 \text{ centimeters}$$

The number zero is not a **positive number,** but it can be used to describe a physical distance of no magnitude. Thus we say that zero is a **real number.** In addition to the positive numbers and zero, in algebra we use numbers that we call **negative numbers,** and these numbers are also called real numbers. The ancients did not understand or use negative numbers. A man could not own negative 10 sheep. If he owned any sheep at all, the number of sheep had to be designated by a number greater than zero. The ancients could subtract 4 from 6 and get 2, but they felt that it was impossible to subtract 6 from 4 because that would result in a number that was less than zero itself. To their way of thinking, this was clearly impossible.

While some might tend to agree with the ancients, to the modern mathematician, physicist, or chemist, the idea or concept of negative numbers does exist, and it is a useful concept. **We say that every positive real number has a negative counterpart, and we call these numbers the *negative real numbers.*** We must always use a minus sign when we designate a negative number, as we see here by writing a negative seven.

$$-7$$

We may use a plus sign to designate a positive number, as we see by writing positive seven.

$$+7$$

Or we may leave off the plus sign as we did in arithmetic and just write the numerical part with no sign.

$$7$$

**We must remember that when we write a numeral with no sign, we designate a positive number.** When we are talking about negative numbers as well as positive numbers, we say that we are talking about **signed numbers.** As we shall see later, the use of signed numbers will enable us to lump the operations of addition and subtraction into a single operation which we will call algebraic addition.

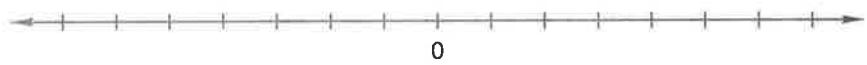
## 4.D number lines

In the 1950s the so-called new math appeared, and among other things it introduced the **number line** at the elementary algebra level. The number line can be used as a graphic aid when discussing signed numbers, and it is especially useful when discussing the addition of signed numbers.

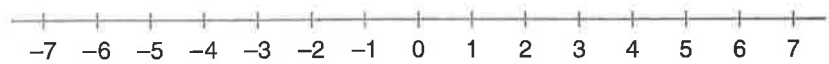
To construct a number line, we first draw a line and divide it into equal units of length. The units may be any length as long as they are all the same length.



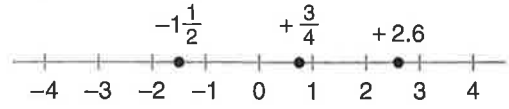
Many books show small arrowheads on the ends of number lines to emphasize that the lines continue without end in both directions, as we show below. The arrowheads are not necessary and may be omitted. Now we choose a point on the line as our base point. We call this base point the **origin,** and we associate the number zero with this point.




Then we associate the positive real numbers with the points to the right of the origin and the negative real numbers with the points to the left of the origin.

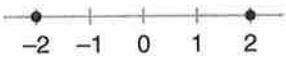



On the number line above we have indicated the location of zero, the counting numbers, and the negative counterpart of each counting number. As required, we can indicate the position of any real number by locating it in relation to the numbers shown. For example, on the number line below we indicate the position of $-1\frac{1}{2}$, $+\frac{3}{4}$, and +2.6 by placing a dot at the approximate location of each number.




**When we place a dot on the number line to indicate the location of a number, we say that we have *graphed* the number and that the dot is the graph of the number. Conversely, the number is said to be the coordinate of the point that we have graphed. We use the number line to tell if one number is greater than another number by saying that a number is *greater* than another number if its graph lies to the right of the graph**

**of the other number.** Thus $\frac{3}{4}$ is greater than $-1\frac{1}{2}$ because the graph of $\frac{3}{4}$ lies to the right of the graph $-1\frac{1}{2}$. This topic will be discussed in considerable detail in later lessons.




On the line above we have graphed +2 and –2. The number +2 (usually the + sign is omitted) lies 2 units to the right of the origin, and the number –2 lies 2 units to the left of the origin. Since the graphs of these numbers are equidistant from the origin but in opposite directions, it is sometimes helpful to think of each of these numbers as being the **opposite** of the other number. In this example, **we say that –2 is the opposite of 2 and that 2 is the opposite of –2.**

## 4.E multiplication and division of fractions

Fractions are multiplied by multiplying the numerators to get the new numerator, and by multiplying the denominators to get the new denominator.

| | PROBLEM | SOLUTION |
|---|---|---|
| (a) | $\frac{4}{3} \times \frac{7}{5}$ | $\frac{4 \times 7}{3 \times 5} = \frac{28}{15} = 1\frac{13}{15}$ |

We divide fractions by inverting the divisor and then multiplying.

| | PROBLEM | INVERTING | SOLUTION |
|---|---|---|---|
| (b) | $\frac{4}{3} \div \frac{15}{8}$ | $\frac{4}{3} \times \frac{8}{15}$ | $\frac{4 \times 8}{3 \times 15} = \frac{32}{45}$ |

If cancellation is possible, it is easier if we cancel before we multiply.

| | PROBLEM | CANCELLATION | SOLUTION |
|---|---|---|---|
| (c) | $\frac{7}{3} \times \frac{30}{9}$ | $\frac{7}{\cancel{3}_{1}} \times \frac{\overset{10}{\cancel{30}}}{9}$ | $\frac{70}{9} = 7\frac{7}{9}$ |
| (d) | $\frac{3}{5} \times \frac{5}{6} \times \frac{21}{23}$ | $\frac{\overset{1}{\cancel{3}}}{\cancel{5}_{1}} \times \frac{\overset{1}{\cancel{5}}}{\cancel{6}_{2}} \times \frac{21}{23}$ | $\frac{21}{46}$ |

We change mixed numbers to improper fractions and then multiply or divide as indicated.

| | PROBLEM | IMPROPER FRACTION | SOLUTION |
|---|---|---|---|
| (e) | $2\frac{1}{2} \times 5\frac{1}{3}$ | $\frac{5}{\cancel{2}_{1}} \times \frac{\overset{8}{\cancel{16}}}{3}$ | $\frac{40}{3} = 13\frac{1}{3}$ |
| (f) | $12\frac{1}{3} \div 2\frac{1}{6}$ | $\frac{37}{\cancel{3}_{1}} \times \frac{\overset{2}{\cancel{6}}}{13}$ | $\frac{74}{13} = 5\frac{9}{13}$ |
| (g) | $\frac{3\frac{1}{3}}{2\frac{1}{5}}$ | $\frac{\frac{10}{3}}{\frac{11}{5}}$ | $\frac{10}{3} \cdot \frac{5}{11} = \frac{50}{33} = 1\frac{17}{33}$ |

## 4.F symbols of equality and inequality

We use the equals sign (=) to designate that two quantities are equal. Thus we can write

$$5 + 2 = 7$$

because the number represented by the notation 5 + 2 is the same number as that represented by the numeral 7. In the same way, we use the symbol ≠ to designate that two quantities are not equal. Thus we can write that

$$5 + 2 \neq 11$$

because 7 is not equal to 11.

## 4.G basic operations

The four basic operations of arithmetic are also the basic operations of algebra. The operations are addition, subtraction, multiplication, and division. We will review these operations here and will restrict our discussion to the numbers of arithmetic, which are the positive real numbers and zero.

### addition

When we wish to add two numbers to get a result, we use the plus sign (+) to indicate the operation of addition. We call each of the numbers an **addend,** and we call the result a **sum.**

$$2 + 3 = 5$$

In this example, we use the plus sign to indicate addition; we say that the numbers 2 and 3 are addends, and we say that 5 is the sum.

**We note that the sum of zero and any particular real number is the particular real number itself.**

$$4 + 0 = 4 \quad \text{and} \quad 15 + 0 = 15$$

### subtraction

When we wish to subtract one number from another number, we use the minus sign (–) to indicate the operation of subtraction. We call the first number the **minuend;** the second number, the **subtrahend;** and the result, the **difference.**

$$9 - 5 = 4$$

In this example, 9 is the minuend, 5 is the subtrahend, and 4 is the difference.

### multiplication

If two numbers are to be multiplied to achieve a result, each of the numbers is called a **factor** and the result is called a **product.** There are several ways to indicate the operation of multiplication.

$$4 \cdot 3 = 12 \qquad 4(3) = 12 \qquad (4) \cdot (3) = 12 \qquad (4)(3) = 12 \qquad 4 \times 3 = 12$$

In each of the five examples shown here, the notation indicates that 4 is to be multiplied by 3 and the result is 12. In algebra, we will avoid the last notation because the cross can be confused with the letter $x$, a symbol we will use for other purposes. In each of the above, we say that 4 and 3 are factors, and we say that 12 is the product.

**We note that the product of a particular real number and the number 1 is the particular real number itself.**

$$4 \cdot 1 = 4 \quad \text{and} \quad 15 \cdot 1 = 15$$

The number zero also has a unique multiplicative property. **The product of any real number and the number zero is the number zero.**

$$4 \cdot 0 = 0 \quad \text{and} \quad 15 \cdot 0 = 0$$

**division** If one number is to be divided by another number to achieve a result, the first number is called the **dividend,** the second number is called the **divisor,** and the result is called the **quotient.**

$$\frac{10}{5} = 2 \qquad 10 \div 5 = 2$$

Both of the notations shown here indicate that 10 is to be divided by 5 and that the result is 2. We call 10 the dividend, call 5 the divisor, and say that the quotient is 2. When the indicated division is expressed in the form of a fraction such as $\frac{10}{5}$, we say that 10 is the **numerator** of the fraction and that 5 is the **denominator** of the fraction.

## 4.H review of operations with decimal numbers

We must align the decimal points vertically when we add and subtract decimal numbers, as we show here.

$$\begin{array}{r} 1.005 \\ +\ 300.012 \\ \hline 301.017 \end{array}$$

example 4.1 Add 4.0016 and 0.02163.

*solution* We remember to place the numbers so that the decimal points are aligned.

$$\begin{array}{r} 4.0016 \\ +\ 0.02163 \\ \hline \mathbf{4.02323} \end{array}$$

example 4.2 Subtract 0.02163 from 4.0016.

*solution* Again we align the decimal points.

$$\begin{array}{r} 4.0016 \\ -\ 0.02163 \\ \hline \mathbf{3.97997} \end{array}$$

example 4.3 Multiply $4.06 \times 0.016$.

*solution* We do not align the decimal points when we multiply.

$$\begin{array}{r} 4.06 \\ \times\ 0.016 \\ \hline 2436 \\ 406 \\ \hline \mathbf{0.06496} \end{array}$$

example 4.4 Divide 6.039 by 0.03.

*solution* As the first step, we adjust the decimal points as necessary. Then we divide.

$$0.03\overline{)6.039} \qquad \begin{array}{r} \mathbf{201.3} \\ 3\overline{)603.9} \\ \underline{6} \\ 3 \\ \underline{3} \\ 9 \\ \underline{9} \end{array}$$

## 4.I unit multipliers

If we multiply a number by a fraction that has a value of 1, we do not change the value of the number. We just change the numeral we use to represent the number. To write 5 with a denominator of 7, we multiply 5 by 7 over 7.

$$\frac{5}{1} \times \frac{7}{7} = \frac{35}{7}$$

The fraction 7 over 7 has a value of 1, so we have just multiplied 5 by 1. Thirty-five over 7 has a value of 5 and is just another way to write 5. The fractions

$$\frac{3 \text{ ft}}{1 \text{ yd}} \quad \text{and} \quad \frac{1 \text{ yd}}{3 \text{ ft}}$$

have units and are equal to 1 because 3 ft is another name for 1 yd. We call these fractions **unit multipliers.** We can use unit multipliers to change the units of a number.

## 4.J conversions of length

In this section we will use unit multipliers to convert length measurements.

example 4.5 Use one unit multiplier to convert 32 feet to inches (1 ft = 12 in.).

*solution* There are two unit multipliers that we can consider.

$$\frac{1 \text{ ft}}{12 \text{ in.}} \quad \text{and} \quad \frac{12 \text{ in.}}{1 \text{ ft}}$$

Let us try the first unit multiplier and see what happens.

$$\frac{32 \text{ ft}}{1} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{32 \text{ ft} \times 1 \text{ ft}}{12 \text{ in.}} = \frac{32 \text{ ft}^2}{12 \text{ in.}}$$

This answer is correct but is not what we want. Let us try the other unit multiplier.

$$\frac{32 \cancel{\text{ft}}}{1} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = \mathbf{32(12)\ in.}$$

We can cancel the ft on the bottom with the ft on top because ft over ft has a value of 1. Since we are not interested in a numerical answer, we will not do the multiplication.

example 4.6 Use one unit multiplier to convert 36 feet to miles (5280 ft = 1 mi).

*solution* There are two unit multipliers that we may consider.

$$\frac{5280 \text{ ft}}{1 \text{ mi}} \quad \text{and} \quad \frac{1 \text{ mi}}{5280 \text{ ft}}$$

We will use the second unit multiplier because the feet on the bottom will cancel the feet on the top. We remember that the symbol $\approx$ means "approximately equals."

$$\frac{36 \cancel{\text{ft}}}{1} \times \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} = \frac{36}{5280} \text{mi} \approx \mathbf{0.0068\ mi}$$

example 4.7 Use one unit multiplier to convert 47.25 inches to centimeters (1 in. = 2.54 cm).

*solution* The inch is **defined** to be **exactly** 2.54 cm, so there are two unit multipliers that we may consider.

$$\frac{1 \text{ in.}}{2.54 \text{ cm}} \quad \text{and} \quad \frac{2.54 \text{ cm}}{1 \text{ in.}}$$

Since we have 47.25 inches, we will use the second unit multiplier because it has inches on the bottom, which will cancel inches on the top.

$$47.25 \text{ in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} = \mathbf{47.25(2.54)\ cm}$$

A numerical answer is not necessary, so we will leave the answer as it is.

example 4.8 Use one unit multiplier to convert 42 meters to centimeters (1 m = 100 cm).

*solution* There are 100 centimeters in 1 meter. The two possible unit multipliers are

$$\frac{100 \text{ cm}}{1 \text{ m}} \quad \text{and} \quad \frac{1 \text{ m}}{100 \text{ cm}}$$

We will use the first unit multiplier because it has meters on the bottom.

$$\frac{42 \text{ m}}{1} \times \frac{100 \text{ cm}}{1 \text{ m}} = \mathbf{4200\ cm}$$

example 4.9 Use two unit multipliers to convert 42 feet to centimeters.

*solution* Many people in the United States still use inches, feet, and miles to make measurements. The rest of the world uses centimeters, meters, and kilometers. Thus, U.S. engineers often find it necessary to convert from one system to another. The crossover point is the **exact relationship** 1 in. = 2.54 cm. We will convert feet to inches and then convert inches to centimeters.

$$42 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} = \mathbf{42(12)(2.54)\ cm}$$

We will not do the multiplication because a decimal answer is not required. We are interested in the method, not in an exact numerical answer.

example 4.10 Use two unit multipliers to convert 4 miles to inches.

*solution* We will convert from miles to feet and then convert feet to inches.

$$4 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = \mathbf{4(5280)(12)\ in.}$$

The following table provides the basic equivalent measures.

TABLE OF EQUIVALENT MEASURES

| | | |
|---|---|---|
| 1 ft = 12 in. | 1 yd = 3 ft | 1 mi = 5280 ft |
| 1 m = 100 cm | 1 cm = 10 mm | 1 km = 1000 m |
| | 1 in. = 2.54 cm | |

**Problems to provide practice in operations with decimal numbers will appear in the problem sets. Do not use a calculator when working these problems.**

**practice** Perform operations as indicated. Do not use a calculator.

**a.** $4\frac{1}{2} \times 2\frac{4}{5}$

**b.** $3\frac{1}{4} \div 1\frac{3}{8}$

**c.** 47.123 + 8.416 + 705.4

**d.** 800.62 − 75.88

**e.** 47.05 × 6.42

**f.** 4.028 ÷ 0.04

**g.** Use two unit multipliers to convert 75 feet to centimeters. (Go from feet to inches to centimeters.)

**h.** Use two unit multipliers to convert 450 inches to miles. (Go from inches to feet to miles.)

## problem set 4

**1.** (4) What is the difference between a number and a numeral?

**2.** (4) (a) What do we call our system of numeration that we use to designate numbers?

(b) Who invented this system?

(c) List the digits that we use in this system.

**3.** (4) (a) What numbers are called counting numbers?

(b) What numbers are called natural numbers?

**4.** (4) The numbers of arithmetic are zero and the positive real numbers. How do we define positive real numbers?

**5.** (2) (a) Define a rectangle.

(b) Define a rhombus.

(c) Define a square.

(d) Is every square also a rectangle?

**6.** (4) Use one unit multiplier to convert 20 inches to centimeters (2.54 cm = 1 in.).

**7.** (4) Use two unit multipliers to convert 25 feet to centimeters. (Go from feet to inches to centimeters.)

**8.** (3) The length of a rectangle is 16 centimeters. The width of the rectangle is 9 centimeters. Find the perimeter of the rectangle.

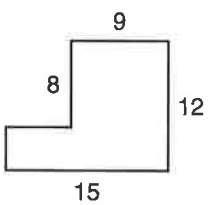
**9.** (3) The perimeter of a square is 24 meters. What is the length of one side of the square?

**10.** (3) The radius of a circle is 8 inches. Find the circumference of the circle.

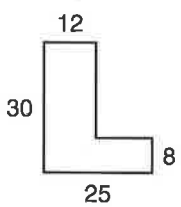
**11.** (3) The diameter of a circle is 10 feet. Find the circumference of the circle.

Find the perimeter of each figure. Corners that look square are square. Dimensions are in centimeters.

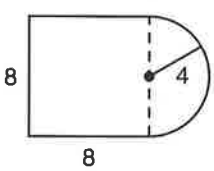
**12.** (3)



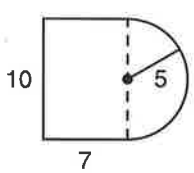

**13.** (3)



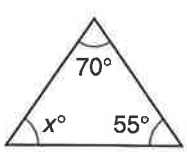

**14.** (3)




**15.** (3)




**16.** (2) Find $x$.



Add, subtract, multiply, or divide as indicated:

**17.** (1) $\frac{17}{24} - \frac{1}{4} - \frac{1}{8}$ **18.** (1) $8\frac{1}{4} + 5\frac{3}{8}$ **19.** (1) $8\frac{1}{4} - 5\frac{3}{8}$

**20.** (1) $95\frac{1}{8} - 4\frac{13}{16}$ **21.** (4) $\frac{4}{3} \times \frac{7}{2} \times \frac{9}{5}$ **22.** (4) $4\frac{1}{2} \times 6\frac{2}{3}$

**23.** (4) $4\frac{1}{2} \div 6\frac{2}{3}$ **24.** (4) $\frac{14\frac{2}{3}}{3\frac{1}{4}}$ **25.** (4) 6.0018 + 0.03121

**26.** (4) 8.0146 − 0.03251 **27.** (4) 16.04 × 3.46 **28.** (4) 5.412 ÷ 0.123

**29.** (1) The length of $\overline{AB}$ is $3\frac{1}{2}$ meters. The length of $\overline{BC}$ is $10\frac{3}{8}$ meters. Find $AC$.




**30.** (1) $PR$ is $15\frac{2}{3}$ kilometers. $PQ$ is $10\frac{1}{6}$ kilometers. Find $QR$.




# *LESSON 5 Sets • Absolute Value • Addition of Signed Numbers*

## 5.A sets

We use the word **set** to designate a well-defined collection of numbers, objects, or things. We say that the individual objects or things that make up the set are the elements of the set or the members of the set. It is customary to designate a set by enclosing the members of the set within braces.

$$A = \{1, 2, 3, 4, 5\}$$

We have designated set $A$ as the set whose members are the counting numbers 1 through 5 inclusive. We could also designate this set by placing a verbal phrase within the braces as

$$A = \{\text{counting numbers 1 through 5 inclusive}\}$$

or we could designate the members of this set by writing a sentence:

Set $A$ is the set whose members are the counting numbers 1 through 5 inclusive.

Since we can designate which numbers are natural or counting numbers so that there is no doubt as to whether a number is or is not a natural or counting number, we say that these numbers constitute a set. Both *natural* and *counting* are names for the same set, and we normally use one name or the other. Thus we say

$$\text{Natural numbers} = \{1, 2, 3, 4, 5, \ldots\}$$

The three dots indicate that the list of numbers continues without end.

**If we include the number zero with the set of natural numbers, we say that we have designated the set of *whole numbers*.**

$$\text{Whole numbers} = \{0, 1, 2, 3, 4, 5, \ldots\}$$

We designate that the list of the members of the set of whole numbers continues without end by using the three dots after the last digit recorded. **Now if we include in our list the negative**