Add, subtract, multiply, or divide as indicated:

17. (1) $\frac{17}{24} - \frac{1}{4} - \frac{1}{8}$ **18.** (1) $8\frac{1}{4} + 5\frac{3}{8}$ **19.** (1) $8\frac{1}{4} - 5\frac{3}{8}$

20. (1) $95\frac{1}{8} - 4\frac{13}{16}$ **21.** (4) $\frac{4}{3} \times \frac{7}{2} \times \frac{9}{5}$ **22.** (4) $4\frac{1}{2} \times 6\frac{2}{3}$

23. (4) $4\frac{1}{2} \div 6\frac{2}{3}$ **24.** (4) $\frac{14\frac{2}{3}}{3\frac{1}{4}}$ **25.** (4) 6.0018 + 0.03121

26. (4) 8.0146 − 0.03251 **27.** (4) 16.04 × 3.46 **28.** (4) 5.412 ÷ 0.123

29. (1) The length of $\overline{AB}$ is $3\frac{1}{2}$ meters. The length of $\overline{BC}$ is $10\frac{3}{8}$ meters. Find AC.



30. (1) PR is $15\frac{2}{3}$ kilometers. PQ is $10\frac{1}{6}$ kilometers. Find QR.



LESSON 5 Sets • Absolute Value • Addition of Signed Numbers

5.A sets

We use the word **set** to designate a well-defined collection of numbers, objects, or things. We say that the individual objects or things that make up the set are the elements of the set or the members of the set. It is customary to designate a set by enclosing the members of the set within braces.

$$A = \{1, 2, 3, 4, 5\}$$

We have designated set A as the set whose members are the counting numbers 1 through 5 inclusive. We could also designate this set by placing a verbal phrase within the braces as

$$A = \{\text{counting numbers 1 through 5 inclusive}\}$$

or we could designate the members of this set by writing a sentence:

Set A is the set whose members are the counting numbers 1 through 5 inclusive.

Since we can designate which numbers are natural or counting numbers so that there is no doubt as to whether a number is or is not a natural or counting number, we say that these numbers constitute a set. Both *natural* and *counting* are names for the same set, and we normally use one name or the other. Thus we say

$$\text{Natural numbers} = \{1, 2, 3, 4, 5, \ldots\}$$

The three dots indicate that the list of numbers continues without end.

If we include the number zero with the set of natural numbers, we say that we have designated the set of *whole numbers*.

$$\text{Whole numbers} = \{0, 1, 2, 3, 4, 5, \ldots\}$$

We designate that the list of the members of the set of whole numbers continues without end by using the three dots after the last digit recorded. **Now if we include in our list the negative**

of every member of the set of natural numbers, we have designated the set that we call the *integers*.

$$\text{Integers} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

The dots on each end indicate that the list continues without end in both directions.

5.B absolute value

The number zero is neither positive nor negative and can be designated with the single symbol 0. Every other real number is either positive or negative and thus requires a two-part numeral. One of the parts is the plus or the minus sign, and the other part is the numerical part. If we look at the two numerals

$$+7 \quad \text{and} \quad -7$$

we note that the numerical part of each one is the same and that the numerals differ only in their signs. We can think of the numerical part as designating the quality of bigness of the number, and we use the words **absolute value** to describe this quality. **However, when we try to write the absolute value of one of these numbers by just writing the numerical part**

$$7$$

we find that we have written a positive number because we have agreed that a numeral written with no sign designates a positive number. Because of this agreement, we are forced to define the absolute value of any nonzero real number to be a positive number. We define the absolute value of zero to be zero. If we enclose a number[†] within vertical lines, we are designating the absolute value of the number. We will demonstrate this notation by designating the absolute value of zero, the absolute value of positive 7, and the absolute value of negative 7.

$|0| = 0$ read "the absolute value of zero equals zero"

$|+7| = +7$ read "the absolute value of 7 equals 7"

$|-7| = +7$ read "the absolute value of –7 equals 7"

Since the plus sign in front of a positive number is customarily omitted, the above can be written without recording the plus signs:

$|7| = 7$ read "the absolute value of 7 equals 7"

$|-7| = 7$ read "the absolute value of –7 equals 7"

Thus we have two rules for stating the absolute value of a real number.

1. **The absolute value of zero is zero.**
2. **The absolute value of any nonzero real number is a positive number.**

Here we designate the absolute value of zero and several other real numbers.

(a) $|0| = 0$ (b) $|-7.12| = 7.12$ (c) $|7.12| = 7.12$

(d) $|-5| = 5$ (e) $|5| = 5$ (f) $\left|\frac{3}{4}\right| = \frac{3}{4}$

[†]We really should use the word *numeral* here, but from now on we will often use the words *number* and *numeral* interchangeably because excessive attention to the difference between these words is counterproductive.

example 5.1 Simplify: (a) $|-5|$ (b) $|11 - 2|$ (c) $-|20 - 2|$

solution (a) The absolute value of –5 is 5.

$$|-5| = \mathbf{5}$$

(b) First we simplify within the vertical lines:

$$|11 - 2| = |9|$$

The absolute value of 9 is 9.

$$|9| = \mathbf{9}$$

(c) Again we simplify within the vertical lines:

$$-|20 - 2| = -|18|$$

The absolute value of 18 is 18, but we want the opposite of this, so our answer is –18.

$$-|18| = \mathbf{-18}$$

5.C addition of signed numbers

In arithmetic the minus sign always means to subtract, but in algebra we also use the minus sign to designate that a number is a negative number. This can be confusing at first, as we see if we look at the expression

$$3 - 2$$

and ask if the minus sign means to subtract or if it means that –2 is a negative number. It turns out that we will find the same answer with either thought process, but in algebra we normally prefer the second process in which we think of the negative sign as designating that –2 is a negative number. If we do this, we say that we are using algebraic addition.

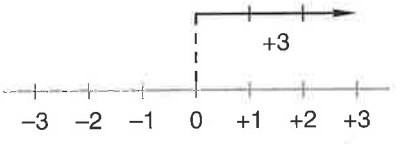
To help explain the rules for algebraic addition, we will use diagrams drawn on a number line.



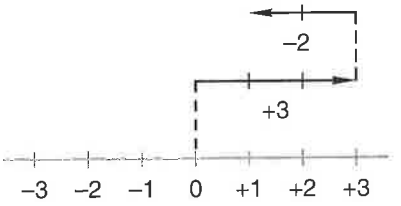
We will represent signed numbers with arrows and say that the arrows indicate directed numbers. We represent positive numbers with arrows that point to the right and negative numbers with arrows that point to the left. The length of each arrow corresponds to the absolute value of the number represented. For instance, +3 and –2 can be represented with the following arrows.



To use these arrows to add +3 and –2, we begin at the origin and draw the +3 arrow pointing to the right.



Then from the head of this arrow we draw the –2 arrow, which points to the left. The head of the –2 arrow is over +1 on the number line.



This is a graphical solution to the problem

$$(+3) + (-2) = +1$$

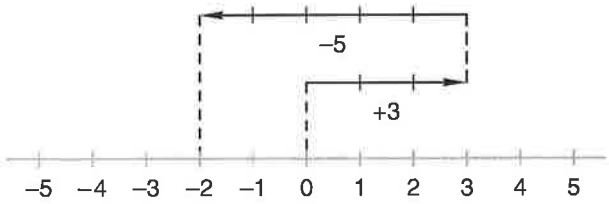
Note that we obtain the same answer when we add signed numbers algebraically as we obtain when we use only the positive numbers of arithmetic and subtract!

$$3 - 2 = 1$$

It may seem that we are trying to turn an easy problem into a difficult problem, but such is not the case. **In algebra the operations of addition and subtraction are lumped together in the one operation of algebraic addition, and this enables a straightforward solution to problems that would be very confusing if the concepts of signed numbers and algebraic addition were not used.**

example 5.2 Use directed numbers and the number line to add +3 and –5.

solution

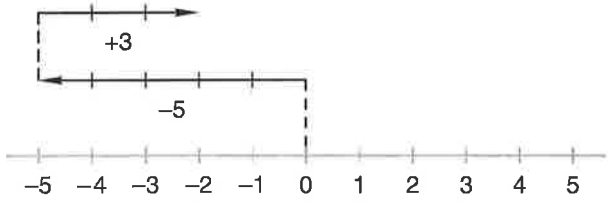


We begin at the origin and draw an arrow 3 units long that points to the right to represent the number +3. From the head of this arrow we draw an arrow 5 units long that points to the left to represent the number –5. The head of the second arrow is just above the number –2 on the number line. Thus we see that

$$(+3) + (-5) = \mathbf{-2}$$

example 5.3 Use directed numbers and the number line to add –5 and +3.

solution



We will use the same arrows, but this time we will draw the –5 arrow first and then draw the +3 arrow. We note that again we get an answer of **–2.**

These examples demonstrate that we may exchange the order in which we add two numbers without changing the answer we get. The Latin word for "exchange" is *commutare*, so we call this peculiarity or property of real numbers the **commutative property for addition.**

COMMUTATIVE PROPERTY FOR ADDITION

The order in which two real numbers are added does not affect the sum. For example,

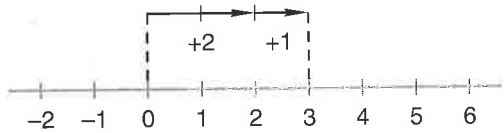
$4 + 3 = 7$ and $3 + 4 = 7$

This property can be used to show that any number of numbers can be added in any order and the answer will be the same every time.

When the signed numbers to be added have the same sign, the arrows will point in the same direction, as we see in the next two examples.

example 5.4 Use directed numbers and the number line to add +2 and +1.

solution

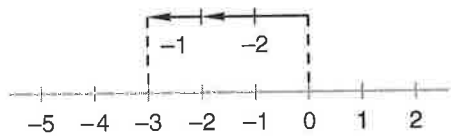


We see from the graph that the solution is +3.

$$(+2) + (+1) = \mathbf{+3}$$

example 5.5 Use directed numbers and the number line to add –2 and –1.

solution



We see from the graph that the solution is –3.

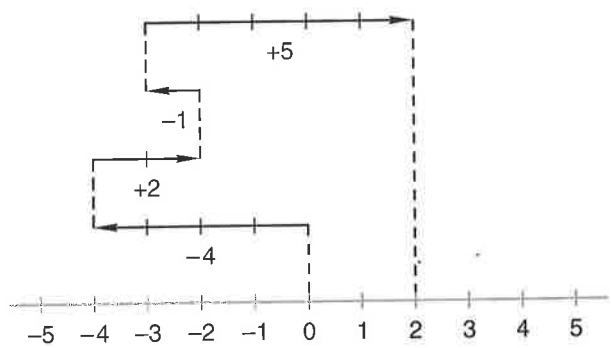
$$(-2) + (-1) = \mathbf{-3}$$

The numbers to be added may also be exchanged when three or more numbers are being added. To demonstrate this we will add four signed numbers, and then exchange the order of the numbers and work the problem again. The sum will be the same.

example 5.6 Use directed numbers and the number line to add these numbers:

$$(-4) + (+2) + (-1) + (+5)$$

solution We will use arrows and add the numbers in the order they are written.

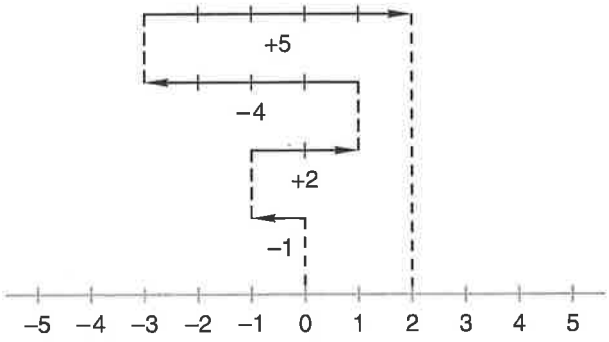


We begin at the origin and move 4 units to the left for –4, then 2 units to the right for +2, then 1 unit to the left for –1, and finally 5 units to the right for +5. We find that we end up directly above the number +2 on the number line. Thus

$$(-4) + (+2) + (-1) + (+5) = \mathbf{+2}$$

The answer will be the same regardless of the order in which we draw the arrows. To show this, we will work the problem again with the order of the numbers changed.

$$(-1) + (+2) + (-4) + (+5)$$



Again we find that the sum of the numbers is +2.

practice

Simplify:

a. $|-4|$ **b.** $|4.2|$ **c.** $-|10 - 6|$ **d.** $-|-14 + 6|$

Draw a number line for each of the following problems and use directed numbers (arrows) to add the signed numbers.

e. $(+3) + (+2)$

f. $(-3) + (+2)$

g. $(-5) + (+2) + (-3) + (+3)$

problem set 5

1. (5) (a) Use braces and digits to designate the set of natural numbers.

(b) Use braces and digits to designate the set of whole numbers.

(c) Use braces and digits to designate the set of integers.

2. (4) What do we call the point on the number line with which we associate the number zero?

3. (4) (a) What is the graph of a number?

(b) What is the coordinate of a point on the number line?

(c) How can we tell if one number is greater than another number?

Simplify:

4. (5) $|-8|$ **5.** (5) $|+8|$ **6.** (5) $|-12|$

7. (5) $-|15 - 5|$ **8.** (5) $-|-15 + 5|$ **9.** (5) $|12 - 30|$

Draw a number line for each of the following problems and use directed numbers (arrows) to add the signed numbers.

10. (5) $(+3) + (-8)$ **11.** (5) $(-1) + (+2)$

12. (5) $(+4) + (+3)$ **13.** (5) $(-4) + (+2) + (-4) + (+8)$

14. (4) Use one unit multiplier to convert 28 centimeters to inches (2.54 cm = 1 in.).

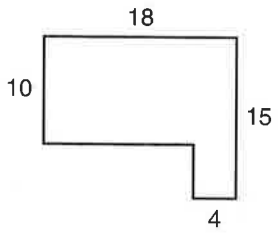
15. (4) Use two unit multipliers to convert 42 centimeters to feet. (Go from centimeters to inches to feet.)

16. (3) The length of a rectangle is 22 inches. The width of the rectangle is 13 inches. Find the perimeter of the rectangle.

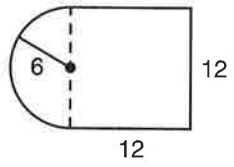
17. (3) The radius of a circle is 10 feet. Find the circumference of the circle.

Find the perimeter of each figure. Corners that look square are square. Dimensions are in yards.

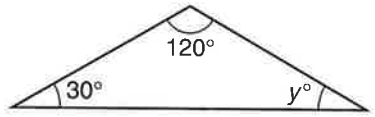
18. (3)



19. (3)



20. (2) Find y.



Add, subtract, multiply, or divide as indicated:

21. (1) $6\frac{2}{3} + 7\frac{4}{9}$

22. (1) $95\frac{1}{8} - 4\frac{13}{16}$

23. (4) $4\frac{1}{2} \times 2\frac{2}{3}$

24. (4) $4\frac{1}{2} \div 7\frac{3}{8}$

25. (4) $\dfrac{7\frac{1}{8}}{4\frac{2}{5}}$

26. (4) $23.0106 + 0.1094$

27. (4) $48.2 - 13.34$

28. (4) 8.08×0.120

29. (4) $8.48636 \div 2.12$

30. (1) XZ is $18\frac{2}{5}$ miles. XY is $6\frac{1}{15}$ miles. Find YZ.



LESSON 6 Rules for Addition • Adding More Than Two Numbers • Inserting Parentheses Mentally • Definition of Subtraction

6.A rules for addition

In the preceding lesson we learned to add signed numbers by using a number line and arrows to represent the numbers. This procedure allows us to have a graphical picture of what we are doing. Unfortunately, this method is slow and time-consuming. We do not have time to go through the entire algebra course drawing number lines and arrows, so we must develop rules that will allow us to do algebraic addition quickly. We need two rules—one to use when the numbers to be added have the same signs and one to use when the numbers have different signs. In the following example we will draw two diagrams that will help us state the first rule.

example 6.1 Use directed numbers and the number line to add +1 and +3 algebraically, and use directed numbers and the number line to add –1 and –3 algebraically.