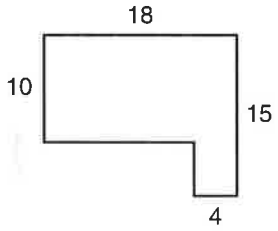
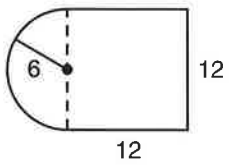
17. (3) The radius of a circle is 10 feet. Find the circumference of the circle.

Find the perimeter of each figure. Corners that look square are square. Dimensions are in yards.

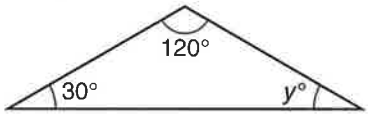
18. (3)



19. (3)



20. (2) Find y.



Add, subtract, multiply, or divide as indicated:

21. (1) $6\frac{2}{3} + 7\frac{4}{9}$

22. (1) $95\frac{1}{8} - 4\frac{13}{16}$

23. (4) $4\frac{1}{2} \times 2\frac{2}{3}$

24. (4) $4\frac{1}{2} \div 7\frac{3}{8}$

25. (4) $\dfrac{7\frac{1}{8}}{4\frac{2}{5}}$

26. (4) $23.0106 + 0.1094$

27. (4) $48.2 - 13.34$

28. (4) 8.08×0.120

29. (4) $8.48636 \div 2.12$

30. (1) XZ is $18\frac{2}{5}$ miles. XY is $6\frac{1}{15}$ miles. Find YZ.



LESSON 6 Rules for Addition • Adding More Than Two Numbers • Inserting Parentheses Mentally • Definition of Subtraction

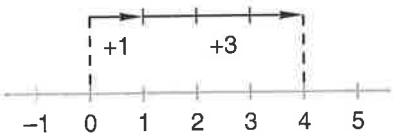
6.A rules for addition

In the preceding lesson we learned to add signed numbers by using a number line and arrows to represent the numbers. This procedure allows us to have a graphical picture of what we are doing. Unfortunately, this method is slow and time-consuming. We do not have time to go through the entire algebra course drawing number lines and arrows, so we must develop rules that will allow us to do algebraic addition quickly. We need two rules—one to use when the numbers to be added have the same signs and one to use when the numbers have different signs. In the following example we will draw two diagrams that will help us state the first rule.

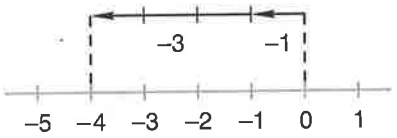
example 6.1 Use directed numbers and the number line to add +1 and +3 algebraically, and use directed numbers and the number line to add –1 and –3 algebraically.

solution

(+1) + (+3)



(−1) + (−3)



We see from these figures that

$$(+1) + (+3) = \mathbf{+4} \qquad \text{and} \qquad (-1) + (-3) = \mathbf{-4}$$

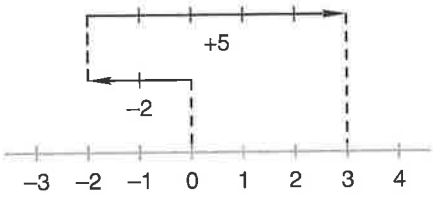
Now we will generalize. **To add algebraically two signed numbers that have the same sign, we add the absolute values of the numbers and give the result the same sign as the sign of the numbers.**

Now we will use two examples in which numbers with different signs are added algebraically to help us state the second rule.

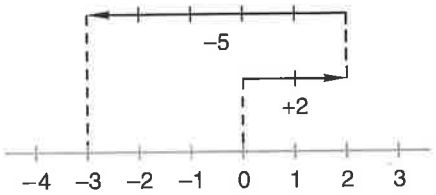
example 6.2 Use directed numbers and the number line to add –2 and +5 algebraically, and use directed numbers and the number line to add +2 and –5 algebraically.

solution

(–2) + (+5)



(+2) + (–5)



From the figure we see that the absolute value of each answer is 3 but that one of the answers is +3 and that one of the answers is –3.

$$(-2) + (+5) = \mathbf{+3} \qquad (+2) + (-5) = \mathbf{-3}$$

In the first case, the number +5 had the larger absolute value and thus the sign of the result was positive. In the second case, the number –5 had the larger absolute value and thus the sign of the result was negative. In both cases, the absolute value of the answer was the difference in the absolute values of the numbers.

Now we will generalize. **To add algebraically two signed numbers that have opposite signs, we take the difference in the absolute values of the numbers and give to this result the sign of the original number whose absolute value is the greatest.**

When two numbers have the same absolute value but different signs, their sum is zero. For instance, the sum of (–5) and (+5) is zero.

$$(-5) + (+5) = 0$$

We say that –5 is the opposite of +5 and that +5 is the opposite of –5. **Every real number except zero has an opposite, and the sum of any real number and its opposite is zero. Another name for the opposite of a number is the additive inverse of the number, so we can also say that the sum of any real number and its additive inverse is zero.**

Test your understanding of the rules by covering the answers to the following problems and seeing if your answers are the same.

(a) $(+7) + (-3) = +4$

(b) $(+2) + (+6) = +8$

(c) $(-7) + (-3) = -10$

(d) $(-2) + (-8) = -10$

(e) $(-7) + (+3) = -4$

(f) $(-2) + (+8) = +6$

(g) $(-4) + (-1) = -5$

(h) $(+2) + (-8) = -6$

6.B adding more than two numbers

We have noted that signed numbers may be added in any order and the answer will not change. Some people add from left to right, and others begin by first adding numbers that have the same sign.

example 6.3 Add: $(-5) + (4) + (-3) + (+2)$

solution This time we will add from left to right.

$$\begin{aligned} (-5) + (4) + (-3) + (+2) &\quad \text{original problem} \\ (-1) + (-3) + (+2) &\quad \text{added } (-5) \text{ and } (4) \\ (-4) + (+2) &\quad \text{added } (-1) \text{ and } (-3) \\ \mathbf{-2} &\quad \text{added } (-4) \text{ and } (+2) \end{aligned}$$

example 6.4 Add: $(-3) + (+2) + (-2) + (+4)$

solution We see that we have two negative numbers and two positive numbers. As the first step, we will add (-3) to (-2) and $(+2)$ to $(+4)$ and then add these sums.

$$\begin{aligned} (-3) + (+2) + (-2) + (+4) &\quad \text{original problem} \\ (-5) + (+6) &\quad \text{added } (-3) \text{ to } (-2) \text{ and } (+2) \text{ to } (+4) \\ \mathbf{+1} &\quad \text{added } (-5) \text{ to } (+6) \end{aligned}$$

6.C inserting parentheses mentally

Most signed number problems are written without parentheses enclosing the signed numbers. We must insert the parentheses mentally before we can add. **We will let the sign preceding the number designate whether the number is a positive number or a negative number, and we will mentally insert a plus sign in front of each number to indicate algebraic addition.** If we use this process,

$4 - 3 + 2$ can be read as $(+4) + (-3) + (+2)$

and

$-6 - 3 - 2 + 5$ can be read as $(-6) + (-3) + (-2) + (+5)$

Thus, to simplify an expression such as

$$-4 + 2 - 3 - 3 - 2 + 6$$

we mentally enclose each of the numbers in parentheses, insert the extra plus signs, and then add.

$$(-4) + (+2) + (-3) + (-3) + (-2) + (+6) = -4$$

Care must be used to avoid associating the signs with the wrong numbers. If the mental parentheses are not used, some would incorrectly read the original expression from right to left as "6 plus 2 minus 3," etc. Guard against this.

example 6.5 Simplify: $-4 - 3 + 2 - 4 - 3 - 2$

solution We mentally enclose each number in parentheses and use plus signs so that we can read the problem as

$$(-4) + (-3) + (+2) + (-4) + (-3) + (-2)$$

Now we add the numbers and get a sum of -14.

$$(-4) + (-3) + (+2) + (-4) + (-3) + (-2) = \mathbf{-14}$$

example 6.6 Simplify: $-2 + 11 - 4 + 3 - 2$

solution We mentally enclose the numbers in parentheses and add algebraically to get a sum of +6.

$$(-2) + (+11) + (-4) + (+3) + (-2) = \mathbf{+6}$$

example 6.7 Simplify: $(-4) + |-2| + 3 - 7 - 2$

solution We mentally insert parentheses so that the problem reads as follows:

$$(-4) + (|-2|) + (+3) + (-7) + (-2)$$

Now we simplify and get an answer of –8.

$$(-4) + (+2) + (+3) + (-7) + (-2) = \mathbf{-8}$$

6.D definition of subtraction

As we have seen, if we use algebraic addition, we can handle minus signs without using the word *subtraction*. We let the signs tell whether the numbers are positive or negative, and we mentally insert parentheses and extra plus signs as necessary. Thus the subtraction problem on the left

$$7 - 4 = 3 \qquad 7 + (-4) = 3$$

can be turned into the algebraic addition problem on the right. A definition of algebraic subtraction does exist, however, and some people prefer to use it rather than using mental parentheses. The result is exactly the same, but the definition uses the word *subtraction*. To subtract algebraically, we change the sign of the subtrahend and add.

$$7 - 4 = 3 \qquad 7 + (-4) = 3$$

The formal definition of the operation of algebraic subtraction is as follows.

> ALGEBRAIC SUBTRACTION
>
> If a and b are real numbers, then
>
> $$a - b = a + (-b)$$
>
> where $-b$ is the opposite of b.

Thus there are two thought processes that may be used to simplify expressions that contain minus signs such as

$$7 - 4$$

Since we prefer to consider that the minus sign designates a negative number, we will emphasize algebraic addition in this book and will avoid the use of the word *subtraction*.

practice Use parentheses to enclose each number or expression and its sign. Then insert plus signs between the parentheses. Then add to get a sum.

a. $-5 - 2 + 7 - 6$

b. $-4 - |-2| - 6 + (-5)$

c. $-|-8| - 3 + 5 - 11$

d. $-8 + |-6| - |5| - 7$

problem set 6

1. (6) State the rule for adding two numbers whose signs are alike.

2. (6) State the rule for adding two numbers whose signs are different.

3. (5) What property of addition states that the order in which two real numbers are added does not affect the sum?

4. (4) (a) What do we call the answer to an addition problem?

(b) What do we call the answer to a subtraction problem?

(c) What do we call the answer to a multiplication problem?

(d) What do we call the answer to a division problem?

Simplify:

5. (5) $|-5|$ **6.** (5) $-|10 - 7|$ **7.** (5) $|3 - 6|$

Draw a number line for each of the following problems and use directed numbers (arrows) to add the signed numbers.

8. (5) $(+4) + (-5)$ **9.** (5) $(+1) + (-3) + (+4) + (-2)$

10. (4) Use one unit multiplier to convert 34 meters to centimeters (100 cm = 1 m).

11. (4) Use two unit multipliers to convert 6 miles to inches. (Go from miles to feet to inches.)

12. (3) The perimeter of a square is 36 centimeters. What is the length of one side of the square?

13. (3) The diameter of a circle is 14 meters. Find the circumference of the circle.

Simplify:

14. (6) $(+3) + (-14)$ **15.** (6) $(-3) + (-14)$

16. (6) $(-5) + (4) + (-3) + (+8)$ **17.** (6) $(-3) + (+2) + (-2) + |-2|$

Use parentheses to enclose each number and its sign. Then insert plus signs between the parentheses. Then add to get a sum.

18. (6) $-2 + 11 - 4 + 3 - 8$ **19.** (6) $-5 - 11 + 20 - 14 + 5$

20. (6) $-4 - 3 + 2 - 4 - 3 - 8$ **21.** (6) $7 - 3 + 2 - 11 + 4 - 5 + 3$

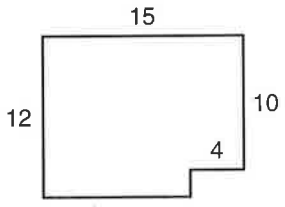
Use parentheses to enclose each number or expression and its sign. Then insert plus signs between the parentheses. Then add to get a sum.

22. (6) $-7 + (-8) + 3$ **23.** (6) $-7 + (-3) + 4 - 3 + (-2)$

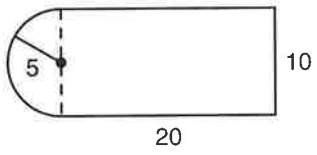
24. (6) $-4 - 2 + (+8) + |-5|$ **25.** (6) $+|-2 - 3| - 4 + (-8)$

Find the perimeter of each figure. Corners that look square are square. Dimensions are in kilometers.

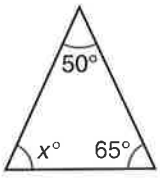
26. (3)



27. (3)



28. (2) Find x.



29. (4) Simplify: $\frac{21}{5} \times \frac{15}{7} \times \frac{4}{9}$

30. (1) AC is $25\frac{3}{4}$ inches. BC is $20\frac{1}{20}$ inches. Find AB.

