18. (7) $-3 - (-3) + |-3|$

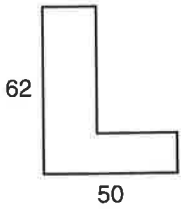
19. (7) $-2 - [-(-6)] + |-5|$

20. (7) $-|-3 - 2| - (-3) - 2 - 5$

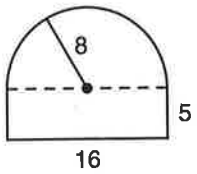
21. (7) $|-2 - 5 - 7| - (-4)$

Find the perimeter of each figure. Corners that look square are square. Dimensions are in yards.

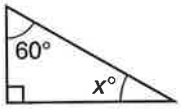
22. (3)



23. (3)



24. (2) Find x.



Add, subtract, multiply, or divide as indicated:

25. (1) $5\frac{1}{2} + 7\frac{3}{8} - 1\frac{1}{4}$

26. (4) $1\frac{3}{5} \times 12\frac{1}{2}$

27. (4) $4\frac{1}{4} \div 3\frac{2}{5}$

28. (4) $0.00143 + 0.012 + 443.6 + 0.0007$

29. (4) 3.628×0.0404

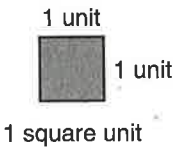
30. (1) The length of $\overline{XY}$ is $16\frac{2}{3}$ centimeters. The length of $\overline{YZ}$ is $5\frac{5}{6}$ centimeters. Find XZ.

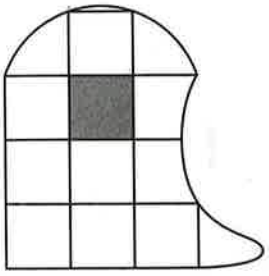


LESSON 8 *Area*

8.A the concept of area

The **area** of a closed figure is the number of square units contained in the figure. A **square unit** is a square having sides that measure one unit in length. Any shape that fills a closed figure without overlapping can be used to measure area, but the square is used because of its simplicity.



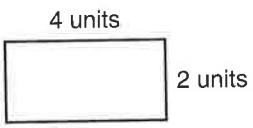


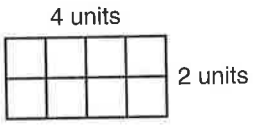
Closed figure with irregular boundary

For many closed figures with irregular boundaries, we must "break up" some of the square units to fill the figure. Therefore, it is not uncommon to have closed figures whose areas contain fractional square units. We can see that counting square units is not the easiest or the best way to find the area of a closed figure. Therefore, in the following sections we will develop formulas for computing the areas of some common geometrical figures.

8.B areas of rectangles and squares

On the left we show a rectangle whose length is 4 units and whose width is 2 units.





Area = 4 units × 2 units
= 8 square units
= 8 units^2

On the right we see that it takes 8 square units to fill the rectangle. Therefore, the area of the rectangle is 8 square units which equals the length of the rectangle times the width of the rectangle. From this we see that the area of a rectangle equals the length times the width.

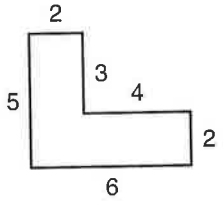
> Area of a rectangle = length × width

A square is a rectangle so the area formula for a rectangle also applies to a square. The length and width of a square are the same, so the area of a square is equal to the square of the length of a side.

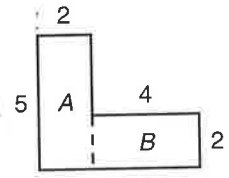
> Area of a square = (length of a side)^2

Areas are always measured in square units. Some common units of area are the square centimeter (cm^2) and the square meter (m^2). It is important to remember that *units* are used for length and *square units* are used for area.

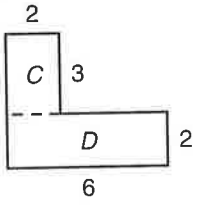
example 8.1 Find the area of this figure. All angles are right angles. Dimensions are in centimeters.



solution We will divide the figure into rectangles and find the area of each of the rectangles. Then we will add the areas. We show two different ways to work this problem.

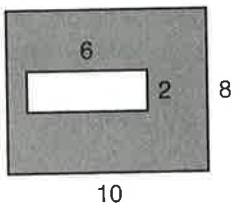


Area A = 2 cm × 5 cm = 10 cm^2
Area B = 4 cm × 2 cm = 8 cm^2
18 cm^2



Area C = 2 cm × 3 cm = 6 cm^2
Area D = 6 cm × 2 cm = 12 cm^2
18 cm^2

example 8.2 Find the area of the shaded portion of this figure. All angles are right angles. Dimensions are in meters.

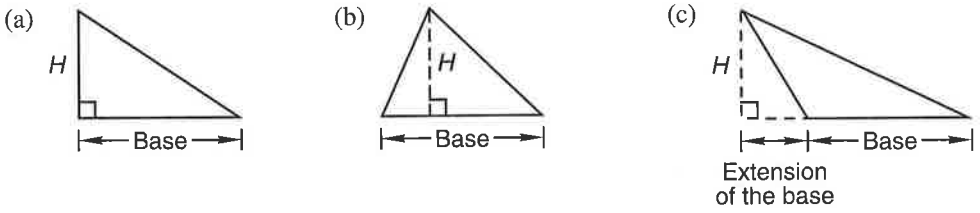


solution The area of the shaded portion of the figure equals the area of the large rectangle minus the area of the small rectangle.

$$\begin{aligned}\text{Area} &= \text{area of large rectangle} - \text{area of small rectangle}\\ &= (10\text{ m})(8\text{ m}) - (6\text{ m})(2\text{ m})\\ &= 80\text{ m}^2 - 12\text{ m}^2\\ &= \mathbf{68\text{ m}^2}\end{aligned}$$

8.C areas of triangles

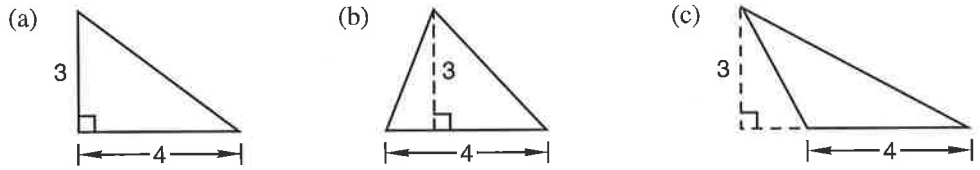
The **altitude,** or **height,** of a triangle is the perpendicular distance from either the base of the triangle or an extension of the base to the opposite vertex. Any one of the three sides can be designated as the base.



The altitude can (a) be one of the sides of the triangle, (b) fall inside the triangle, or (c) fall outside the triangle. When the altitude falls outside the triangle, we have to extend the base so that the altitude can be drawn. This extension of the base is not part of the length of the base. The area of any triangle equals one half the product of the base and the height.

$$\text{Area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

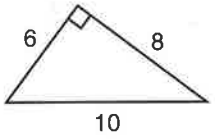
example 8.3 Find the areas of these triangles. Dimensions are in inches.



solution The base of each triangle is 4 in. and the height of each triangle is 3 in. Thus, all the triangles have the same area.

$$\text{Area} = \frac{B \times H}{2} = \frac{4\text{ in.} \times 3\text{ in.}}{2} = \mathbf{6\text{ in.}^2}$$

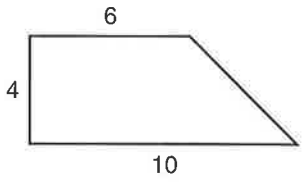
example 8.4 Find the area of this right triangle. Dimensions are in feet.



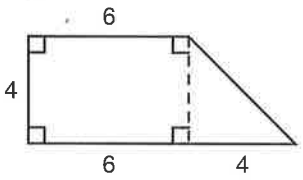
solution Any one of the three sides of the right triangle can be designated as the base. We will not choose the side of length 10 ft as the base because it would be too difficult to find the height. We can choose either of the two remaining sides as the base. The other side then becomes the height.

$$\text{Area} = \frac{1}{2}BH = \frac{1}{2}(6\text{ ft})(8\text{ ft}) = \mathbf{24\text{ ft}^2}$$

example 8.5 Find the area of this figure. Corners that look square are square. Dimensions are in yards.



solution First we divide the figure into a rectangle and a right triangle.



The area of the figure equals the sum of the areas of the rectangle and the right triangle.

$$\begin{aligned}\text{Area} &= \text{area of rectangle} + \text{area of right triangle}\\ &= (6\text{ yd})(4\text{ yd}) + \frac{1}{2}(4\text{ yd})(4\text{ yd})\\ &= 24\text{ yd}^2 + 8\text{ yd}^2\\ &= \mathbf{32\text{ yd}^2}\end{aligned}$$

8.D areas of circles

We remember that the radius of a circle is the distance from the center of the circle to any point on the circle. The area of a circle equals πr^2, where r is the length of the radius of the circle.



Area = πr^2

Area of a circle = πr^2

example 8.6 The radius of a circle is 3 centimeters. Find the area of the circle.

solution The formula for the area of a circle of radius r is given by

$$\text{Area} = \pi r^2$$

Substituting 3 cm for r, we get

$$\begin{aligned}\text{Area} &= \pi(3\text{ cm})^2\\ &= (3.14)(9\text{ cm}^2)\\ &= \mathbf{28.26\text{ cm}^2}\end{aligned}$$

example 8.7 The area of a circle is 25 square meters. Find the radius of the circle.

solution The formula for the area of a circle of radius r is given by

$$\text{Area} = \pi r^2$$

Substituting 25 m^2 for area, we get

$$25\ m^2 = \pi r^2$$

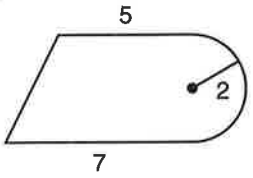
$$25\ m^2 = (3.14)r^2$$

$$7.96\ m^2 = r^2$$

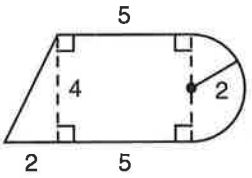
Solving for r, we get

$$r = \mathbf{2.82\ m}$$

example 8.8 Find the area of this figure. Lines that look parallel are parallel. Dimensions are in inches.



solution First we divide the figure into a right triangle, a rectangle, and a semicircle (half of a circle).



The area of the figure equals the sum of the areas of the right triangle, the rectangle, and the semicircle.

$$\text{Area} = \text{area of right triangle} + \text{area of rectangle} + \text{area of semicircle}$$

$$= \frac{1}{2}(2\text{ in.})(4\text{ in.}) + (5\text{ in.})(4\text{ in.}) + \frac{\pi(2\text{ in.})^2}{2}$$

$$= 4\text{ in.}^2 + 20\text{ in.}^2 + \frac{(3.14)(2\text{ in.})^2}{2}$$

$$= 24\text{ in.}^2 + 6.28\text{ in.}^2$$

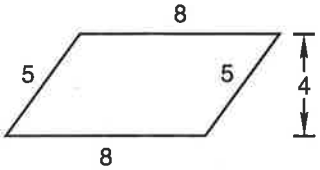
$$= \mathbf{30.28\ in.^2}$$

These problems are arithmetic problems. A calculator can be used to help with the arithmetic. **We remember that in this book, we will use 3.14 as an approximation for π when doing calculations that involve the number π.**

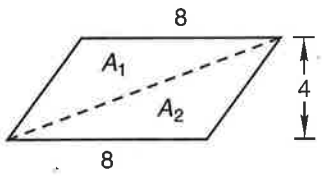
8.E areas of parallelograms and trapezoids

There is a formula for the area of a parallelogram. There is also a formula for the area of a trapezoid. These formulas are hard to remember because they are used so seldom. The easiest way to find the areas is to divide the figures into two triangles as we show in the following examples.

example 8.9 Find the area of this parallelogram. Dimensions are in centimeters.



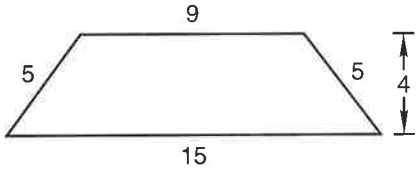
solution A diagonal of a parallelogram divides the figure into two triangles whose areas are equal. The base of each triangle is 8 cm and the height of each triangle is 4 cm.



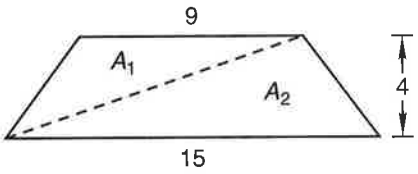
The area of the parallelogram equals the sum of the areas of the two triangles.

$$\begin{aligned}\text{Area} &= A_1 + A_2 \\ &= \frac{1}{2}(8\text{ cm})(4\text{ cm}) + \frac{1}{2}(8\text{ cm})(4\text{ cm}) \\ &= 16\text{ cm}^2 + 16\text{ cm}^2 \\ &= \mathbf{32\text{ cm}^2}\end{aligned}$$

example 8.10 Find the area of this trapezoid. Dimensions are in meters.



solution A diagonal of a trapezoid divides the figure into two triangles. The base of one triangle is 9 m and the base of the other triangle is 15 m. The height of each triangle is 4 m.

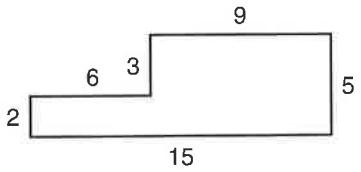


The area of the trapezoid equals the sum of the areas of the two triangles.

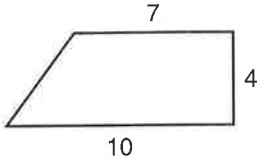
$$\begin{aligned}\text{Area} &= A_1 + A_2 \\ &= \frac{1}{2}(9\text{ m})(4\text{ m}) + \frac{1}{2}(15\text{ m})(4\text{ m}) \\ &= 18\text{ m}^2 + 30\text{ m}^2 \\ &= \mathbf{48\text{ m}^2}\end{aligned}$$

practice

a. Find the area of this figure. All angles are right angles. Dimensions are in inches.

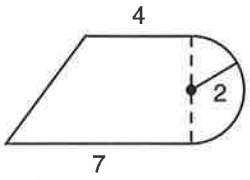


b. Find the area of this figure. Corners that look square are square. Dimensions are in feet.



c. The radius of a circle is 5 centimeters. Find the area of the circle.

d. Find the area of this figure. Corners that look square are square. Dimensions are in meters.



problem set 8

1. (6,7) (a) What is the opposite of $\frac{1}{2}$?

(b) What is the opposite of $-\frac{1}{2}$?

(c) What is the sum of a real number and its opposite?

2. (6) What is another name for the opposite of a number?

3. (2) What angles are formed by lines that are perpendicular?

4. (2) What angle is formed by two right angles?

5. (4) Use two unit multipliers to convert 36 feet to centimeters. (Go from feet to inches to centimeters.)

6. (4) Use two unit multipliers to convert 44 inches to meters. (Go from inches to centimeters to meters.)

7. (3) The perimeter of a rectangle is 30 centimeters. The length of the rectangle is 10 centimeters. Find the width of the rectangle.

8. (8) The length of a rectangle is 5 meters. The width of the rectangle is 4 meters. Find the area of the rectangle.

9. (3) The radius of a circle is 3 inches. Find the circumference of the circle.

10. (8) The radius of a circle is 4 feet. Find the area of the circle.

Use the concept of opposites and algebraic addition to simplify the following. Use additional plus signs and brackets as required.

11. (7) $-(-4) + (-2) - (-3)$

12. (6) $-3 + (-3) + (-6) - 2$

13. (6) $-7 + 3 - 2 - 5 + (-6)$

14. (7) $5 - 3 - (-2) - [-(-3)]$

15. (7) $-|-2| - (-2)$

16. (7) $-|-2| + |2| - (-2)$

17. (6) $7 - 4 - 5 + 12 - 2 - |-2|$

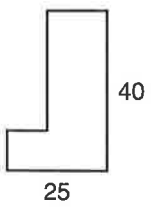
18. (7) $|-4 - 3| - 2 + 7 - (-3)$

19. (7) $5 - |-2 + 5| - (-3) + 2$

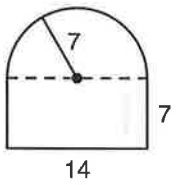
20. (7) $4 - 3 - (-2) - |12 - 3 + 4|$

Find the perimeter of each figure. Corners that look square are square. Dimensions are in yards.

21. (3)

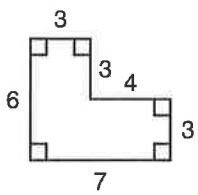


22. (3)

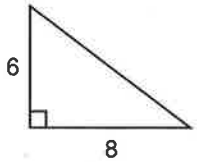


Find the area of each figure. Dimensions are in centimeters.

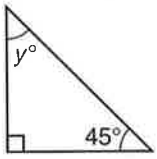
23. (8)



24. (8)



25. (2) Find y.



Add, subtract, multiply, or divide as indicated:

26. (1) $8\frac{1}{18} - 2\frac{1}{6} - 4\frac{1}{3}$

27. (4) $3\frac{2}{3} \times 1\frac{4}{5} \times 2\frac{3}{11}$

28. (4) $4.016 + 0.984$

29. (4) $\frac{0.0832}{4.16}$

30. (1) PR is $33\frac{1}{3}$ meters. PQ is $5\frac{5}{6}$ meters. Find QR.



LESSON 9 *Rules for Multiplication of Signed Numbers • Inverse Operations • Rules for Division of Signed Numbers • Summary*

9.A rules for multiplication of signed numbers

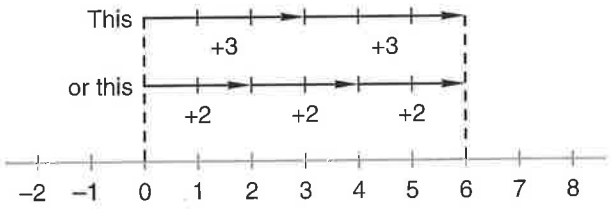
The sum of three 2s is 6. Also the sum of two 3s is 6.

$$2 + 2 + 2 = 6 \qquad 3 + 3 = 6$$

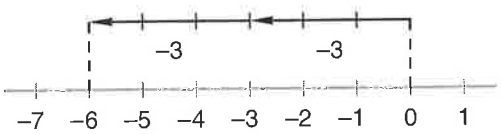
We can get the same results from multiplication by writing

$$3 \cdot 2 = 6 \quad \text{or} \quad 2 \cdot 3 = 6$$

because multiplication is just a shorthand notation for repeated addition of the same numeral. Thus, if we wish to use the number line to explain the multiplication of $3 \cdot 2$, we can do it two ways. We can show the sum of two 3s or the sum of three 2s.



Now let us find the product of 2 and –3 on the number line. We can obtain the same answer by adding two –3s.



Thus we see that

$$(-3) + (-3) = -6 \qquad \text{so} \qquad 2 \cdot (-3) = -6$$