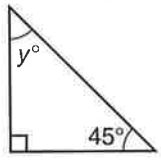
25. (2) Find y.



Add, subtract, multiply, or divide as indicated:

26. (1) $8\frac{1}{18} - 2\frac{1}{6} - 4\frac{1}{3}$

27. (4) $3\frac{2}{3} \times 1\frac{4}{5} \times 2\frac{3}{11}$

28. (4) $4.016 + 0.984$

29. (4) $\frac{0.0832}{4.16}$

30. (1) PR is $33\frac{1}{3}$ meters. PQ is $5\frac{5}{6}$ meters. Find QR.



LESSON 9 Rules for Multiplication of Signed Numbers • Inverse Operations • Rules for Division of Signed Numbers • Summary

9.A rules for multiplication of signed numbers

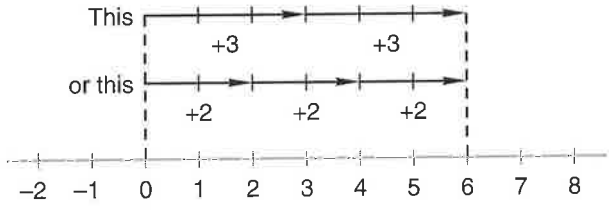
The sum of three 2s is 6. Also the sum of two 3s is 6.

$$2 + 2 + 2 = 6 \qquad 3 + 3 = 6$$

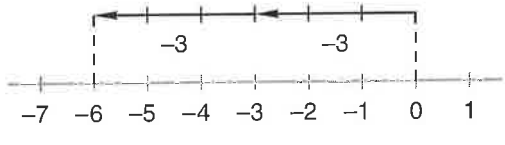
We can get the same results from multiplication by writing

$$3 \cdot 2 = 6 \quad \text{or} \quad 2 \cdot 3 = 6$$

because multiplication is just a shorthand notation for repeated addition of the same numeral. Thus, if we wish to use the number line to explain the multiplication of $3 \cdot 2$, we can do it two ways. We can show the sum of two 3s or the sum of three 2s.



Now let us find the product of 2 and –3 on the number line. We can obtain the same answer by adding two –3s.



Thus we see that

$$(-3) + (-3) = -6 \qquad \text{so} \qquad 2 \cdot (-3) = -6$$

But now if we attempt to use the number line to show –3 times 2 by trying to draw –3 arrows that are +2 units long, we find that the task is impossible. We do not know how to draw –3 arrows because any number of arrows we draw will be a number greater than or equal to 1.

The number line was a useful graphic aid in understanding the concept of signed numbers and their algebraic addition but is of less help when discussing the multiplication or division of signed numbers, so we will not try to use it further for this purpose. Instead we will state the rules for multiplication and division of signed numbers and show some examples.

The three rules for multiplication of signed numbers are the following:

1. The product of two positive real numbers is a positive real number whose absolute value is the product of the absolute values of the two numbers. The following examples demonstrate this rule:
   (a) $(+3)(+4) = 12$ (b) $2(9) = 18$ (c) $4 \cdot 5 = 20$
2. The product of two signed real numbers that have opposite signs is a negative real number whose absolute value is the product of the absolute values of the two numbers. The following examples demonstrate this rule:
   (a) $(-2)(4) = -8$ (b) $6(-2) = -12$ (c) $(-3)(5) = -15$
3. The product of two negative real numbers is a positive real number whose absolute value is the product of the absolute values of the two numbers. The following examples demonstrate this rule:
   (a) $-2(-3) = 6$ (b) $(-5)(-3) = 15$ (c) $-4(-5) = 20$

9.B inverse operations

If one operation will *undo* another operation, the two operations are called *inverse operations*. If we take a particular number and then add and subtract the same number, the result is the particular number itself. For example, if we begin with the number 7 and add 3 and then subtract 3, the result is 7.[†]

$$7 + 3 - 3 = 7$$

Thus addition and subtraction are inverse operations.

Multiplication and division are also inverse operations. If we multiply 7 by 2 and then divide by 2, the result is 7.

$$\frac{7 \cdot 2}{2} = 7$$

There has been no change since dividing by 2 *undoes* the effect of multiplying by 2.

Since multiplication and division are inverse operations, our rules for the multiplication and division of signed numbers must be stated in such a way that these operations are inverse operations.

9.C rules for division of signed numbers

The four rules for division of signed numbers are the following:

1. If one positive real number is divided by another positive real number, the quotient is a positive real number whose absolute value is the quotient of the absolute values of the original numbers. The following examples demonstrate this rule:
   (a) $\frac{6}{3} = 2$ (b) $\frac{8}{2} = 4$ (c) $\frac{12}{4} = 3$

[†]Now we use algebraic addition instead of subtraction.

2. If a negative real number is divided by a positive real number, the quotient is a negative real number whose absolute value is the quotient of the absolute values of the original numbers. The following examples demonstrate this rule:

(a) $\frac{-6}{2} = -3$ (b) $\frac{-10}{5} = -2$ (c) $\frac{-12}{3} = -4$

3. If a positive real number is divided by a negative real number, the quotient is a negative real number whose absolute value is the quotient of the absolute values of the original numbers. The following examples demonstrate this rule:

(a) $\frac{6}{-2} = -3$ (b) $\frac{10}{-5} = -2$ (c) $\frac{12}{-3} = -4$

4. If one negative real number is divided by another negative real number, the quotient is a positive real number whose absolute value is the quotient of the absolute values of the original numbers. The following examples demonstrate this rule:

(a) $\frac{-6}{-3} = 2$ (b) $\frac{-8}{-2} = 4$ (c) $\frac{-12}{-4} = 3$

9.D summary

We can consolidate all we have learned about the multiplication and division of signed numbers into two rules:

1. ***Like signs.*** **The product or the quotient of two signed numbers that have the same sign is a positive number whose absolute value is the absolute value of the product or the quotient of the absolute values of the original numbers.**
2. ***Unlike signs.*** **The product or the quotient of two signed numbers that have opposite signs is a negative number whose absolute value is the absolute value of the product or the quotient of the absolute values of the original numbers.**

We can state the rules above in a less rigorous but more easily remembered way if we say

IN BOTH MULTIPLICATION AND DIVISION

1. *Like* signs $\xrightarrow{yield}$ a positive number.
2. *Unlike* signs $\xrightarrow{yield}$ a negative number.

practice

Simplify the following expressions by using the rules for multiplication and division of signed numbers.

a. $-4(2)$ **b.** $3(-2)$ **c.** $-4(+3)$ **d.** $(-3)(-5)$

e. $\frac{4}{2}$ **f.** $\frac{-6}{3}$ **g.** $\frac{8}{-4}$ **h.** $\frac{-16}{-2}$

problem set 9

1. (9)

(a) Is the product or the quotient of two signed numbers that have the same sign a positive number or a negative number?

(b) Is the product or the quotient of two signed numbers that have opposite signs a positive number or a negative number?

2. (6,7) (a) What is the additive inverse of 3?

(b) What is the additive inverse of –3?

(c) What is the sum of a real number and its additive inverse?

3. (6) What is another name for the additive inverse of a number?

Simplify the following expressions by using the rules for multiplication and division of signed numbers.

4. (9) (2)(5)

5. (9) –5(+2)

6. (9) 5(–2)

7. (9) (–3)(–5)

8. (9) –(2)(–3)

9. (9) $\frac{6}{3}$

10. (9) $\frac{-18}{3}$

11. (9) $\frac{8}{-4}$

12. (9) $\frac{-16}{-2}$

13. (4) Use two unit multipliers to convert 320 centimeters to feet. (Go from centimeters to inches to feet.)

14. (4) Use two unit multipliers to convert 65 meters to inches. (Go from meters to centimeters to inches.)

15. (3) The perimeter of a rectangle is 40 inches. The width of the rectangle is 8 inches. Find the length of the rectangle.

16. (8) The area of a square is 9 square feet. What is the length of one side of the square?

17. (8) The radius of a circle is 5 yards. Find the area of the circle.

Use the concept of opposites and algebraic addition to simplify the following. Use additional plus signs and brackets as required.

18. (7) $-6 - 4 - (3) - (-3) + 3$

19. (7) $-6 + (-3) - [-(-2)] + 7$

20. (7) $-|-6| - [-(-2)] + 5$

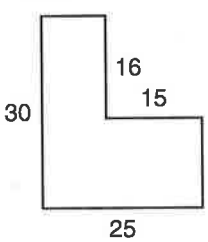
21. (7) $-7 - 4 - (-3) + |-3|$

22. (7) $-3 + (-3) - (-5) - |7|$

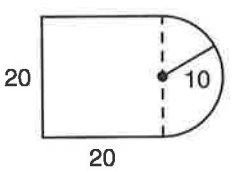
23. (6) $-|-5 + 3 - 2| + 2$

Find the perimeter of each figure. Corners that look square are square. Dimensions are in centimeters.

24. (3)

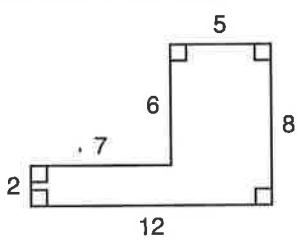


25. (3)

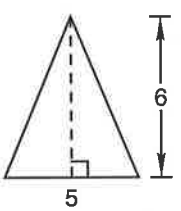


Find the area of each figure. Dimensions are in meters.

26. (8)



27. (8)



Simplify:

28. (4) 52.3 – 15.26

29. (4) 4.03 × 0.220

30. (1) AC is $20\frac{5}{12}$ kilometers. BC is $6\frac{1}{4}$ kilometers. Find AB.

