Name _____

Teacher Note:

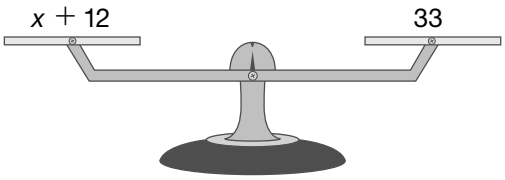
- Refer students to "Algebraic Addition" on page 12 and "Properties of Equality" on page 23 in the *Student Reference Guide*.

page 559

Focus on

• Balanced Equations

- All equations with an equals sign are **balanced equations.** The left side of the equals sign is exactly the same amount as the right side of the equals sign.
- This investigation will show a formal, algebraic method for solving equations.
- The picture of the scale shows that each side of the equation is equal or "balanced." To keep a scale or an equation balanced, add exactly the same amount to both sides, or remove exactly the same amount from both sides.



- There are two steps to balancing an equation.

  Step 1: *Select the operation* (+, −, ×, ÷) that will isolate the variable.

  "Isolating the variable" means to leave only the letter on one side of the equation.

  To select the operation, use the **opposite** of the operation shown.

  Subtraction and addition are opposites.

  Multiplication and division are opposites.

  Step 2: *Perform the operation* on *both* sides of the equation.

- As a third step, check the answer. Do this by replacing the variable in the original equation with the new answer. If the equation balances, then the answer is correct.

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Example

x **+ 12 = 33**

1. *Select the operation.*

In $x + 12 = 33$ we select subtraction because subtracting 12 will isolate the variable *x.*

2. *Perform the operation.*

To keep the scale balanced, subtract 12 from both sides of the equation.

$x + 12 - 12 = 33 - 12$

$x = 33 - 12 = 21$

After subtracting, x is isolated on one side of the equation and 21 balances x on the other side of the equation.

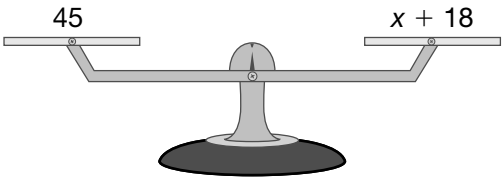
This shows that $x = 21$.

3. Now *check* the solution by replacing x with 21 in the original equation.

$x + 12 = 33$	original equation
$21 + 12 = 33$	replaced x with 21
$33 = 33$	simplified left side

Since *both* sides of the equation equal 33, the solution $\boldsymbol{x = 21}$ is correct.

- Use a balance scale to illustrate equation $45 = x + 18$.



- This time, the unknown number (variable) is on the right side of the balance scale.

1. What operation will isolate the variable? _____

2. Describe how to perform the operation and keep the scale balanced.

S_____ 18 from b_____ sides of the e_____.

3. What will remain on the left and right sides of the balance scale after the operation is performed?

On the l_____ side will be _____, and on the r_____ side will be _____.

- You can check the solution by replacing x with 27 in the original equation.

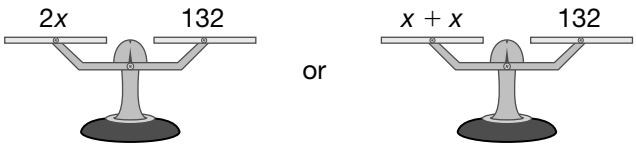
$45 = x + 18$	original equation
$45 = 27 + 18$	replaced x with 27
$45 = 45$	simplified right side

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4. If you reverse the two trays of the balance scale so that $x + 18$ appears on the left side and 45 appears on the right side, would the scale still be balanced? _____ Write the new equation.

$x +$ _____ $=$ _____

- Now look at an equation that has multiplication rather than addition: $2x = 132$.
- Since $2x$ means 2 x's ($x + x$), the equation can be shown on a balance scale two ways.



1. *Select the operation.*
To isolate the *x,* use the opposite operation.
The opposite of multiplication is division.
So, to isolate *x,* divide by 2.

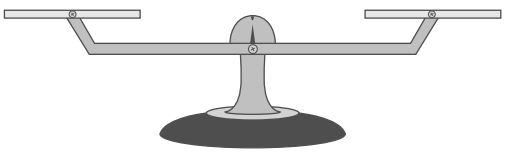
2. *Perform the operation.*
To keep the equation balanced, divide both sides by 2.
Here is a line-by-line solution of the equation.

$2x = 132$	original equation
$\frac{2x}{2} = \frac{132}{2}$	divided both sides by 2
$1x = 66$	simplified both sides
$x = 66$	$1x = x$

3. *Now check the solution.*

$2x = 132$	original equation
$2 \times 66 = 132$	replaced x with 66
$132 = 132$	simplified left side

5. Use this balance scale to show the equation $3x = 132$.



6. What operation will isolate the variable?

D_____ by _____.

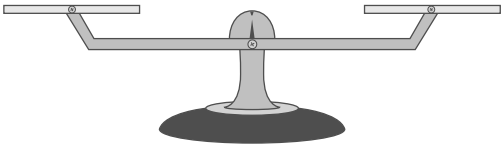
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7. Describe how to perform the operation in a way that will keep the scale balanced.

D_____ both sides of the e_____

by _____.

8. Use this balance scale to show what is on both sides of the scale after the operation is performed.



Math Language

A **coefficient** is the number that multiplies the variable in an algebraic expression. The coefficient is 1 if no number is specified.

9. Write a line-by-line solution of this equation.

$3x$ = _____

$\frac{3x}{3}$ = _____

x = _____

10. Show the check of the solution.

$3x$ = _____

3×44 = _____

132 = _____

- Most students solve the equation $3x = 132$ by dividing both sides of the equation by 3. There is another way to solve the equation.
- The **coefficient** of x is the number that multiplies x. In the equation $3x = 132$, 3 is the coefficient of x.
- To solve the equation, instead of dividing by the coefficient of x, multiply by the reciprocal of the coefficient. So, instead of dividing by 3, multiply by $\frac{1}{3}$.

$$3x = 132$$

$$\frac{1}{3} \times 3x = \frac{1}{3} \times 132$$

$$1x = \frac{132}{3}$$

$$x = 44$$

- For a whole number and decimal coefficients, it is easier to divide by the coefficient. For fractions, it is easier to multiply by the reciprocal of the coefficient.

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- Use the following equation for questions **11–14.** Notice that the coefficient is a fraction.

$$\frac{3}{4}x = \frac{9}{10}$$

11. Select the operation that will result in $\frac{3}{4}x$ becoming $1x$ in the equation. Tell why you chose the operation.

Reciprocal

M _____ by — because it is the

r _____ of —.

12. What operation will keep the equation balanced?

M _____ both sides of the equation by —.

13. Write a line-by-line solution of the equation.

The answer will be an improper fraction.

$$\frac{3}{4}x = \frac{9}{10}$$
$$\frac{4}{3} \times \frac{3}{4}x =$$
$$1x =$$
$$x =$$

14. Show the check of the solution.

$$\frac{3}{4}x =$$
$$\frac{3}{4} \times \frac{6}{5} =$$
$$\frac{18}{20} =$$
$$\frac{9}{10} =$$

- For each of the following equations:
  - **a.** State the operation selected to isolate the variable.
  - **b.** Tell what operations you will perform to keep the equation balanced.
  - **c.** Write a line-by-line solution of the equation.
  - **d.** Show the check of the solution.

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15. $x + 2.5 = 7$

a. S _____ 2.5.

b. S _____ 2.5 from b _____ sides of the equation.

c. $x + 2.5 = 7$

$x + 2.5 - 2.5 = 7 - $ _____

$x =$ _____

d. _____ $+ 2.5 = 7$

$7 = 7$

16. $3.6 = y + 2$

a. S _____ 2.

b. S _____ 2 from _____ sides of the equation.

c. $3.6 = y + 2$

$3.6 -$ _____ $= y + 2 -$ _____

_____ $=$ _____

d. $3.6 =$ _____ $+ 2$

$3.6 =$ _____

17. $4w = 132$

a. D _____ by _____.

b. _____ both s _____ of the equation by _____.

c. $4w = 132$

$\frac{4w}{} = \frac{132}{}$

$w =$ _____

d. $4 \times$ _____ $= 132$

_____ $= 132$

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Use pages 563–564 in your textbook to complete problems **18–22.**